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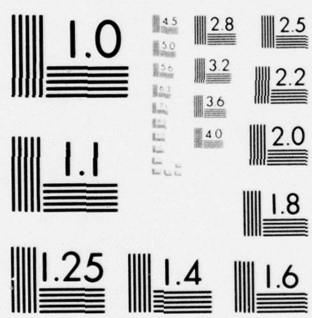

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OPTIMIZATION IN MILITARY PERSONNEL MANAGEMENT

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The computer-based optimization methodology discussed in this paper was developed for the Bureau of Naval Personnel for the purpose of defining long-range management goals for each major Navy personnel skill category. These goals are defined in terms of objective optimum force distributions by skill grouping, by length of service, and by paygrade. The objective function minimized in the optimization methodology is cost per utile per man-year. Function evaluation involves the use of five major models: the Per Capita Cost Model, the Stable Strength Model, the Utility Model, the Reenlistment

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Elasticity Model, and the Optimization Methodology. Descriptions and examples illustrating the models and the overall optimization methodology are presented.

FOREWORD

This study was jointly sponsored by the Navy Personnel Research and Development Center and the Office of Naval Research, and was conducted in support of the Bureau of Naval Personnel. The vehicle for this work was provided by the Office of Naval Research under contract number N00014-72-C-0526. Special acknowledgment is due to Mr. Robert H. Lehto, BUPERS-2X, who fathered many of the concepts underlying this application of optimization methodology to personnel planning, and provided both operational support and substantive guidance for this work. Acknowledgments are also due to Mr. Thomas A. Blanco and Dr. Calvin B. Lee, who technically reviewed and edited this report.

J. J. CLARKIN
Commanding Officer

SUMMARY

Problem

An important problem in Navy personnel planning and management is specifying an "ideal" or "optimum" force distribution. The problem is complicated in that the cost, stability, and utility of an enlisted man vary greatly by skill grouping, length of service, and paygrade.

Past attempts to specify an ideal force distribution have made assumptions about some of the key variables involved in force definition. For example, assumptions have been made about the "correctness" of historical continuation rates or the "reasonableness" of certain rates of advancement. It should be appreciated that, in general, making these assumptions a priori in effect predefines the limits of the ideal or optimum force so that ideal force becomes simply a quantification of a priori biases. In the development of the optimization methodology, the philosophy has been to reduce to an absolute minimum reliance on a priori assumptions, and introduce force distribution constraints only in response to limitations observed in the optimum solution.

Objective

The objective was to develop a computer-based optimization methodology to help determine optimum force distributions by skill grouping, length of service, and paygrade. The objective function minimized in the optimization methodology is cost per utile per man-year for the total Navy enlisted force.

Approach

Optimum force distributions were determined through the development and integration of five major models:

1. Steady-State Inventory Projection Model--a model that calculates the equilibrium force distribution rating, paygrade, and LOS as a function of the number of jobs to be done, the expected continuance behavior, advancement parameters, and the input mix by initial length of enlistment.

2. Per Capita Cost Model--a total budgetary cost model (including retirement costs) that develops the unit cost of personnel by rating, paygrade, and LOS as a function of force parameters developed by the force strength projection models.

3. Utility Model--a model that expresses quantitatively (on a numerical scale from 0 to 100) the accrual of value to the Navy, by the average enlisted man serving at a given paygrade and LOS.

4. Elasticity Dependency Model--a model that estimates the cost to the Navy of changing the prevailing reenlistment rates of the enlisted force.

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5. Optimization Methodology--an algorithm that searches over the range of policy variables for enlisted personnel inventory distributions satisfying personnel and cost constraints at least cost per unit utility.

These five models were linked together to arrive at a total enlisted force distribution with a minimum cost per utile per man-year, which satisfies all the personnel policy constraints specified.

Findings

Exercise of the optimization methodology for each of 87 Navy ratings has been completed. A $97.6 million cost-benefit advantage of the ALNAV optimum force distribution over the feasible force distribution was found. The feasible inventory distribution illustrates that, over the long term (steady state), the observed continuance behavior of the enlisted force will support only a 33.7 percent career ratio. The optimizer, by adjusting the continuance rates for each rating, increased the steady state career ratio to 35.66 percent. By way of comparison, the actual ALNAV career ratio is approximately 43 percent. Other significant differences between the optimized inventory and the feasible inventory include:

1. Increase in LOS 4 continuance rate from .35 to .38.

2. Decrease in LOS 10 through 19 continuance rates, each by .01 or .02.

3. Increase in TOPSIX ratio from 59.44 percent to 61.07 percent.

4. A younger enlisted inventory, and particularly a younger careerist inventory.

5. A greater number of E-5s, E-6s, and E-7s.

6. An increase in average paygrade from 3.990 to 4.056.

7. A decrease in number of retirees from 6131 to 6009 annually.

Conclusions

1. By minimizing cost per utile, the model allows the amount of cost and utility to vary. Although the optimum force had a higher utility than the feasible force, the cost of the optimum force was over $87M more.

2. The model optimizes each skill rating separately and consequently, suboptimizes over all ratings. However, a model for all skill ratings taken together is too large to solve computationally on a computer.

3. The initial results of the model have been positively accepted at the Bureau of Naval Personnel. The model is being implemented as a personnel planning and management tool at BUPERS.

Recommendations

It is recommended that other objective functions besides the present one of minimizing cost per utile be investigated. One alternative objective function could maximize the utility of the enlisted force subject to a constant budget. A second alternative objective function could minimize the cost of the enlisted force subject to a utility constraint.

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Problem

An important problem in Navy personnel planning and management is specifying an "ideal" or "optimum" force distribution. The problem is complicated in that the cost, stability, and utility of an enlisted man vary greatly by skill grouping, length of service, and paygrade.

Past attempts to specify an ideal force distribution have made assumptions about some of the key variables involved in force definition. For example, assumptions are made about the "correctness" of historical continuation rates or the "reasonableness" of certain rates of advancement. It should be appreciated that, in general, making these assumptions a priori in effect predefines the limits of the ideal or optimum force so that the ideal force becomes simply a quantification of a priori biases.

Ideally, two conditions should exist for the development of truly optimum force distributions. First, it is necessary to have a rigorously stated objective function upon which mathematical operations can be performed to determine either the minimum or maximum value of some significant variable or set of variables. Technically, "optimum" cannot be used as a concept unless this condition exists. Second, the explicit values of none of the parameters should be assumed a priori but, rather, should be treated as true variables subject to manipulation in search for an optimum solution. This last condition is, in a practical sense, impossible in a manpower/personnel analysis because of the impossibility of separating the empirically true data from the contextual effects of the historical environment. Further, in terms of practical management, the tolerance of the system for change must also be considered. For example, if a particular rating at some LOS has an historical continuance rate of 90 percent, it would be very difficult to lower it to 50 percent in the short run simply to meet the demands for mathematical optimization. In the development of the optimization methodology, the philosophy has been to reduce to an absolute minimum reliance on a priori assumptions and to introduce force distribution constraints only in response to limitations observed in the optimum solution.

Objective

The objective of this effort was to develop a computer-based optimization methodology to help determine optimum force distributions by skill grouping, length of service, and paygrade. The objective function minimized in the optimization methodology is cost per utile per man-year for the total Navy enlisted force.

APPROACH

Optimum force distributions were determined through the development and integration of five major models: (1) Steady-State Inventory Model, (2) Per Capita Cost Model, (3) Utility Model, (4) Elasticity Dependency Model, and (5) Optimization Methodology. These five models are linked together to arrive at a total enlisted force distribution with a minimum cost per utile per man-year, which satisfies all the personnel policy constraints specified.

Steady-State Inventory Projection Model

The Steady-State Inventory Projection Model (ASTATIC) (Buckley, 1972) is a time-independent model which depicts equilibrium force structures as a function of force size, input mix, continuance rates, and advancement policy. The model is structured, such that, by varying the values of the input parameters, it may be used to determine the objective structure of individual ratings, groups of ratings, or the Navy as a whole. Each such aggregation is defined in terms of the following parameters:

1. Size of personnel community (by rating or for total Navy).

2. Loss behavior dependent upon length of service.

3. Loss behavior for the first 9 years dependent upon the length of initial enlistment, e.g., 2, 3, 4, or 6-year initial enlistment.

4. Variance in loss behavior dependent upon paygrade.

5. Percentage of new input enlisting for 2, 3, 4, and 6-years.

6. Percentage of new input attriting at or immediately following recruit training.

7. Over-the-zone advancement opportunity for each paygrade.

8. Distribution of advancement opportunity by year, within the advancement zone for each paygrade.

9. Beginning year of advancement zone and length of zone for each paygrade.

The ASTATIC model develops an equilibrium force distribution individually for each rating as a function of these rating parameters. Since the force is at equilibrium, in each year the entire inventory in each paygrade LOS cell must leave and be replaced. With the exception of LOS 1 and LOS 2, there are two ways for the replacement inventory to enter a given cell: (1) age in--from the same paygrade, next lower LOS cell; or (2) advance in--from the next lower paygrade, next lower LOS cell. There are three ways for the inventory to leave a given cell: (1) leave the rating, (2) age out--to the same paygrade, next higher LOS cell, or (3) advance out--to the next higher paygrade, next higher LOS.

The equilibrium relationship:

Inventory Aging In + Inventory Advancing In

=

Losses + Inventory Aging Out + Inventory Advancing Out

is solved by the model for each LOS and paygrade cell. The model starts as if there were no inventory in the rating. Beginning with LOS 1, the losses and advancements are computed for each year of the 31-year time frame. At each LOS, losses are first computed and distributed, followed by advancement and aging computations. Loss behavior for the first 9 years of service is defined in the model in terms of continuance rates by obligor type; that is, by initial contract length. Overall continuance rates are computed as a function of the mix of obligor types making up the nonprior service inputs to the rating. These overall continuance rates are combined with the total rating inventory to compute the inventory at each LOS.

Distribution of the losses at each LOS to the paygrades at that LOS is defined by the loss-propensity factors. These factors specify the relationship that the number of losses to a given paygrade-LOS cell bears in proportion to the number of losses to other paygrades at that same LOS.

Advancement behavior is defined in the model in terms of these sets of parameters:

1. Advancement zone definition--begin LOS and end LOS of the zone in which advancements are permitted to the next higher grade.

2. Over-the-zone opportunity--the probability that a man entering a given paygrade will, if not attrited, advance to the next higher paygrade at same LOS within the advancement zone.

3. Advancement distributions--control the allocation of advancements to each LOS cell within the zone. The model assumes that advancements are normally distributed within the zone so that the mean and standard deviation of the advancement distribution are specified for each paygrade.

Outputs of the model include Begin Strength, Net Strength, and Advancements matrices (LOS by paygrade). These force distribution matrices are further processed by the Per Capita Cost Model, the Utility Model, and the Optimization Model.

An example Steady-State Inventory for the Boatswains Mates (BM) rating is illustrated in Table 1. Individual rating statistics by paygrade displayed below the inventory table are as follows:

1. Percentage--the percentage of the rating total inventory which is represented by that paygrade.

2. Careerists--the inventory in that paygrade which has completed at least 4 years of service.

3. Retirements--the number of losses in that paygrade which have completed at least 20 years of service.

4. TIS Advancement (OUT)--the average time in service when advancement out of the paygrade occurs.

5. TIS Serving--the average time in service of the inventory serving in the paygrade.

6. Annual Advancement Opportunity--the percentage of that grade which advanced to the next higher grade.

7. Zone Advancement Opportunity--the probability of being advanced from that grade at some time within the zone of advancement.

8. Zone--the minimum and maximum years of service for advancement in and advancement out, respectively, for that paygrade.

9. Nonprior Service Input--the total amount of new inventory required to support the total rating inventory. This total is further broken down in terms of the number of 2, 3, 4, and 6-year contracts required.

Per Capita Cost Model

The Per Capita Cost Model (PCM) (BUPERS, 1972) is an outgrowth of the billet cost concepts as developed for the Secretary of the Navy's Task Force on Personnel Retention (1965) (Department of the Navy, 1966) and includes most military manpower costs to the U.S. Government from procurement to the end of retirement. The per capita costs are the monies that are being, or will be, expended on an average man in all grades and in all possible years of service for all general ratings and apprenticeships.

The PCM is designed to compute the actual cost of an enlisted man (by rating, LOS, grade) using the ADSTAP system (Silverman, 1971) as the source of personnel inventory distribution and statistical data and the BCM Data Base as the source of basic cost data elements for each of the general and service ratings. The annualized per capita cost in a given rating is computed against the ADSTAP begin and end inventory and advancements distribution as determined by existing or simulated policies. An example output per capita cost matrix is shown in Table 2.

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Table 1

Steady State Inventory Matrix for BM Rating

LOS	E1	E2	E3	E4	E5	E6	E7	E8	E9	Total
1	0.0	0.0	2306.00	0.0	0.0	0.0	0.0	0.0	0.0	2306.00
2	0.0	0.0	2605.00	0.0	0.0	0.0	0.0	0.0	0.0	2605.00
3	0.0	0.0	248.68	2164.32	0.0	0.0	0.0	0.0	0.0	2413.00
4	0.0	0.0	185.46	104.49	1731.05	0.0	0.0	0.0	0.0	2021.00
5	0.0	0.0	52.40	27.34	491.26	0.0	0.0	0.0	0.0	571.00
6	0.0	0.0	34.35	17.92	107.68	216.04	0.0	0.0	0.0	376.00
7	0.0	0.0	33.35	17.52	79.95	238.18	0.0	0.0	0.0	369.00
8	0.0	0.0	30.70	16.61	76.55	229.15	0.0	0.0	0.0	353.00
9	0.0	0.0	23.26	14.53	69.97	211.24	0.0	0.0	0.0	319.00
10	0.0	0.0	16.13	13.84	67.46	200.52	0.0	0.0	0.0	298.00
11	0.0	0.0	10.44	11.85	60.33	65.38	120.00	0.0	0.0	268.00
12	0.0	0.0	8.04	10.16	58.46	45.93	132.40	0.0	0.0	255.00
13	0.0	0.0	6.21	9.62	56.84	44.55	127.78	0.0	0.0	245.00
14	0.0	0.0	4.33	9.00	55.43	43.82	124.41	0.0	0.0	237.00
15	0.0	0.0	3.02	8.43	54.14	43.22	94.87	27.32	0.0	231.00
16	0.0	0.0	2.60	8.16	53.22	42.85	70.55	50.61	0.0	228.00
17	0.0	0.0	2.05	7.46	50.91	41.92	54.19	51.94	12.53	221.00
18	0.0	0.0	1.84	7.34	49.80	41.79	46.31	44.82	72.10	219.00
19	0.0	0.0	1.52	6.85	48.42	41.04	42.27	36.58	37.31	214.00
20	0.0	0.0	0.76	4.60	39.31	33.96	34.64	26.90	35.82	176.00
21	0.0	0.0	0.08	0.77	21.48	21.13	21.37	13.69	23.48	102.00
22	0.0	0.0	0.03	0.28	13.99	14.98	14.50	8.66	15.57	68.00
23	0.0	0.0	0.02	0.16	9.39	10.88	9.73	5.76	10.08	46.00
24	0.0	0.0	0.01	0.10	7.39	5.55	7.98	4.53	8.44	34.00
25	0.0	0.0	0.01	0.04	5.31	4.42	6.36	3.74	7.12	27.00
26	0.0	0.0	0.00	0.04	3.69	3.28	4.93	3.08	5.98	21.00
27	0.0	0.0	0.00	0.03	2.98	2.65	3.98	2.54	4.82	17.00
28	0.0	0.0	0.00	0.03	2.73	2.46	2.89	2.38	4.51	16.00
29	0.0	0.0	0.00	0.02	2.22	2.02	2.51	2.13	4.09	13.00
30	0.0	0.0	0.00	0.02	1.84	1.77	2.51	1.53	3.63	11.00
31	0.0	0.0	0.00	0.02	1.43	1.26	1.42	0.88	2.00	7.00
32	0.0	0.0	5576.28	2461.56	3223.23	1609.98	925.31	287.08	202.46	14285.89
PERCENTAGE			39.0334	17.2307	22.5623	11.2697	6.4771	2.0096	1.4172	
CAREERISTS			231.	193.	1492.	1610.	925.	287.	202.	4941.
RETIREMENTS			2.	7.	48.	41.	41.	31.	44.	214.
TIS ADVANCEMENT			2.0026	3.0013	5.1103	10.1382	15.2286	17.2953	0.0	
TIS SERVING			1.4506	3.1380	6.3925	10.1010	14.5826	17.9794	20.6155	
ANNUAL ADV. OPP.			0.4337	0.8560	0.1399	0.0920	0.0900	0.1746	0.0	
ZONE ADV. OPP.			0.9000	0.9500	0.7501	0.7500	0.6500	0.7000		
ZONE			2.-8.	3.-10.	5.-16.	10.-22.	14.-26.	16.-28.		

NON PRIOR SERVICE INPUT: 3117 148. 2 YEAR 0. 3 YEAR 2970. 4 YEAR 0. 6 YEAR

Table 2

Illustrative Per Capita Annual Cost Matrix

Year	Rating								
	E-1	E-2	E-3	E-4	E-5	E-6	E-7	E-8	E-9
1	12569.	13306.	13624.	10980.	11483.	12669.	13963.	17820.	20421.
2	9574.	10311.	10630.	10763.	11495.	12681.	13746.	17603.	20204.
3	9018.	9755.	10397.	11238.	12205.	13391.	14421.	17603.	20204.
4	9048.	9785.	10685.	12228.	12955.	13688.	14794.	17633.	20234.
5	9039.	9776.	10932.	12581.	13058.	14031.	15118.	17624.	20225.
6	9039.	9776.	10932.	12298.	12769.	13956.	15118.	17624.	20225.
7	9039.	9776.	10932.	12573.	13282.	14280.	15461.	17624.	20225.
8	9039.	9776.	10932.	12476.	13254.	14247.	15461.	17624.	20225.
9	9039.	9776.	10932.	12354.	13379.	14489.	15784.	17624.	20225.
10	9039.	9776.	10932.	12354.	13360.	14489.	15808.	17624.	20225.
11	9039.	9776.	10932.	12354.	13702.	14828.	16590.	18219.	20225.
12	9039.	9776.	10932.	12354.	13702.	14828.	16471.	18219.	20225.
13	9039.	9776.	10932.	12354.	14026.	15327.	16465.	18299.	20558.
14	9039.	9776.	10932.	12354.	14026.	15327.	16460.	18299.	20558.
15	9039.	9776.	10932.	12354.	14201.	15655.	16969.	18802.	21062.
16	9039.	9776.	10932.	12354.	14201.	15655.	16969.	18802.	21062.
17	9039.	9776.	10932.	12354.	14201.	15988.	17297.	18980.	21247.
18	9039.	9776.	10932.	12354.	14201.	15899.	17297.	18980.	21247.
19	9039.	9776.	10932.	12354.	14201.	16159.	17635.	19302.	21591.
20	9039.	9776.	10932.	12354.	14201.	16159.	17635.	19302.	21591.
21	9039.	9776.	10932.	12354.	14201.	16159.	17801.	19645.	21904.
22	9039.	9776.	10932.	12354.	14201.	16159.	17801.	19645.	21904.
23	9039.	9776.	10932.	12354.	14201.	16159.	18643.	20478.	22740
24	9039.	9776.	10932.	12354.	14201.	16159.	18643.	20478.	22740.
25	9039.	9776.	10932.	12354.	14201.	16159.	18643.	20478.	22740.
26	9039.	9776.	10932.	12354.	14201.	16159.	18643.	20478.	22740.
27	9039.	9776.	10932.	12354.	14201.	16159.	20313.	22156.	24327.
28	9039.	9776.	10932.	12354.	14201.	16159.	20313.	22156.	24327.
29	9039.	9776.	10932.	12354.	14201.	16159.	20313.	22156.	24327.
30	9039.	9776.	10932.	12354.	14201.	16159.	20313.	22156.	24327.
31	9039.	9776.	10932.	12354.	14201.	16159.	20313.	22156.	24327.

Table 3 lists the cost elements and sources maintained in the data base. There are two categories of per capita cost: (1) Base Per Capita Costs and (2) Distributed Annualized Costs. The Basc Per Capita Costs are the cost elements that are resident in the data bank that can be fully reconstituted into the per capita format. Navy-wide cost elements consist of four matrices: base pay, hazardous duty pay, FICA (government contribution), and Navy-wide constant cost by grade and year. Both base pay and hazardous duty pay are maintained in a 9 x 31 matrix format. The other basic constant costs, grade- or year-dependent, are extended into the per capita cost by addition to the appropriate vector. FICA (the government's contribution) is also computed and added to the matrix elements.

In using PCM as a planning and projection model, base pay and retirement pay may be modified by an escalation rate. Navy-wide constant costs by grade and by year may also be escalated at different rates as a function of the base pay escalation rate. These rates are used as multipliers of the base pay escalation rate and are then applied to the respective elements. In the absence of rate inputs, a default value of zero is assumed.

The Distributed Annualized Costs are the cost elements that are resident in the data bank in a cost vector format. The aggregate of these costs on an annualized basis is distributed to the appropriate year and grade cost cells as a function of the ADSTAP inventory distribution and advancement matrices. These costs include the uniform allowance, reenlistment bonus, hazardous duty pay, school costs, and retirement costs.

Training costs are input to the model by rating and by LOS. These include basic training and A, B, and C school costs.

The Per Capita Cost Model provides several methods for computing the retirement assessment:

1. Budget--retired Navy enlisted annual expenditure.

2. Normalized Force--projected retirement consumption of the rating flow.

3. Retirement Force--computation of the expected retirement cost of the inventory losses from LOS 20 through 31.

Retirement assessments for each rating may then be distributed to each grade and length of service cell in proportion to the population, either as a function of base pay or as a function of the probability of retirement. Any of these methods of assessment or distribution of retirement costs may be appropriate, depending on the particular application intended by the user.

As employed in the optimization procedure, expected retirement costs are distributed to each LOS-paygrade cell as a function of the probability of retirement of the personnel associated with that cell.

Table 3

BCM Data Element Budget Category and Source

Data Element	MPN	O&MN	Other Budget
Base Pay	x		
LOS dep. Cost			
Clothing Alws			
Initial	x		
Basic	x		
Std	x		
Command & Admin.		x	
Commissary			MILCON
Death Gratuity	x		
Disability Pay	x		
Exchange			MILCON
Interest on Deposits	x		
Insurance Housing			DoD (TFR)
Medical Costs		x	
Messing Subsist	x		
Oversea Sta Alw	x		
Prisoner Appreh.	x		
Recreation Fac.			MILCON
Quarters Single			DoD - MC, OP
Quarters Married			DoD - MC, OP
Servicemens Group Life Insurance	x		
Severance	x		
Unemployment			Dod (TFR)

Data Element	MPN	O&MN	Other Budget
Grade dep. Cost			
Dependent School		x	
Family Sep Alw	x		
Quarters Alw	x		
Sew & Foreign Duty	x		
Tuition Assist.		x	
Hazard Pay	x		
Pro-Pay	x		
Enlisted School Costs		x	
Officer School Costs			
Travel (includes accession, trg, separation, operational, rotational, organized units)	x		
Reenlistment Bonus	x		
FICA	x		
E-7 Clothing Alw	x		
Procurement Personnel		x	
Retirement			DoD
Responsibility Pay	x		
Continuation Pay	x		
Settlement Costs	x		

Utility Model

The Utility Model expresses quantitatively (on a numerical scale from 0 to 100) the accrual of value to the Navy of the average enlisted man serving at a given paygrade and LOS. The utility matrix implemented within ADSTAP is shown in Table 4. Results were derived from a series of experiments using questionnaires and Delphi techniques to solicit opinions of Navy experts from headquarters as well as the Fleet (Schmid & Hovey, 1976).

Example Analysis Employing the Inventory, Cost, and Utility Models

An example analysis performed using the Inventory, Cost, and Utility models will illustrate some of their applications. Suppose the planner is interested in exploring the cost benefits of an increase or decrease in the continuance of the force into the career portion of the Navy; that is, continuing into the fifth year of service. Continuance is a construct of a number of actions by the enlisted force: reenlistment, extension, and serving out a prior enlistment. Continuance rate is a common terminology quantifying these actions. The fourth-year continuance rate is the percentage of the men in a given year with 4 years of service who continue into the following year with an additional year of service. In this case, the parameter of interest to the planner--continuance rate--is an explicit input parameter to the cost and strength models. (Of course, the planner might well be interested in evaluating the impact of increasing reenlistment rate alone. In this case he would need to make some transformation of reenlistment rates into equivalent continuance rates for use in the model exercise.)

In constructing this example, a number of runs were made for each of the following continuance rates in LOS 4: 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0. The remainder of the inputs defining the set of continuance, advancement, input mix, cost elements, etc. was already available within the integrated data base as a default set of parameters. Given this default set, it was only necessary to supply those inputs for the parameters to be varied and to identify the rating. For this example, the BM rating with an enlisted population of 15,983 men was chosen.

Figure 1 presents the change in petty officer ratio (or Top Six ratio) as a function of continuance rate. Note that the ratio varies from about 48 percent to 78 percent as the continuance rate varies from 0.0 to 1.0. Depending on the particular interests of the planner, this may not be acceptable. In the past, the policy of the Navy placed the Top Six ratio at 65 percent so that, if this policy were to prevail, the cost, strength, and utility measures are not really comparable. In order to hold to the 65 percent ratio over the range of continuance rates, changes in advancement policy must be accompanied simultaneously with changes in continuance behavior. The advancement parameter selected for this purpose was the over-the-zone advancement opportunity provided to the E-3; that is, the probability of an E-3 advancing to E-4 at some time within the zone of advancement.

Table 4

Utility Matrix Implemented Within ADSTAP

Year	E-1	E-2	E-3	E-4	Paygrade E-5	E-6	E-7	E-8	E-9
1	.	8.	16.	26.	38.	47.	54.	58.	62.
2	7.	14.	20.	29.	40.	48.	55.	59.	63.
3	7.	15.	29.	35.	43.	50.	56.	60.	64.
4	7.	14.	31.	42.	48.	53.	58.	61.	65.
5	7.	14.	31.	45.	54.	57.	60.	63.	66.
6	6.	14.	31.	46.	59.	61.	63.	65.	68.
7	6.	13.	31.	47.	62.	66.	66.	67.	70.
8	5.	13.	31.	46.	64.	70.	70.	70.	72.
9	5.	12.	30.	46.	65.	73.	74.	73.	75.
10	4.	11.	30.	46.	65.	75.	77.	77.	78.
11	3.	10.	30.	46.	65.	76.	81.	80.	81.
12	3.	9.	29.	45.	65.	77.	83.	83.	84.
13	2.	8.	28.	45.	65.	78.	85.	86.	86.
14	1.	7.	28.	44.	64.	78.	86.	88.	89.
15	1.	6.	27.	43.	64.	78.	87.	90.	91.
16	1.	5.	26.	42.	63.	78.	88.	91.	93.
17	1.	4.	24.	40.	62.	78.	89.	92.	95.
18	.	3.	23.	38.	61.	78.	89.	93.	96.
19	.	2.	22.	37.	59.	77.	89.	94.	97.
20	.	2.	20.	35.	57.	76.	89.	94.	98.
21	.	1.	19.	33.	55.	75.	89.	94.	98.
22	.	1.	18.	31.	52.	73.	89.	95.	99.
23	.	1.	16.	29.	49.	71.	88.	95.	99.
24	.	1.	15.	27.	47.	68.	87.	95.	100.
25	.	.	14.	25.	44.	65.	85.	95.	100.
26	.	.	14.	24.	42.	61.	82.	94.	100.
27	.	.	13.	23.	41.	58.	78.	93.	100.
28	.	.	12.	22.	39.	56.	74.	89.	100.
29	.	.	12.	21.	38.	54.	71.	85.	98.
30	.	.	11.	21.	37.	53.	69.	81.	91.
31	.	.	11.	20.	37.	52.	68.	79.	87.

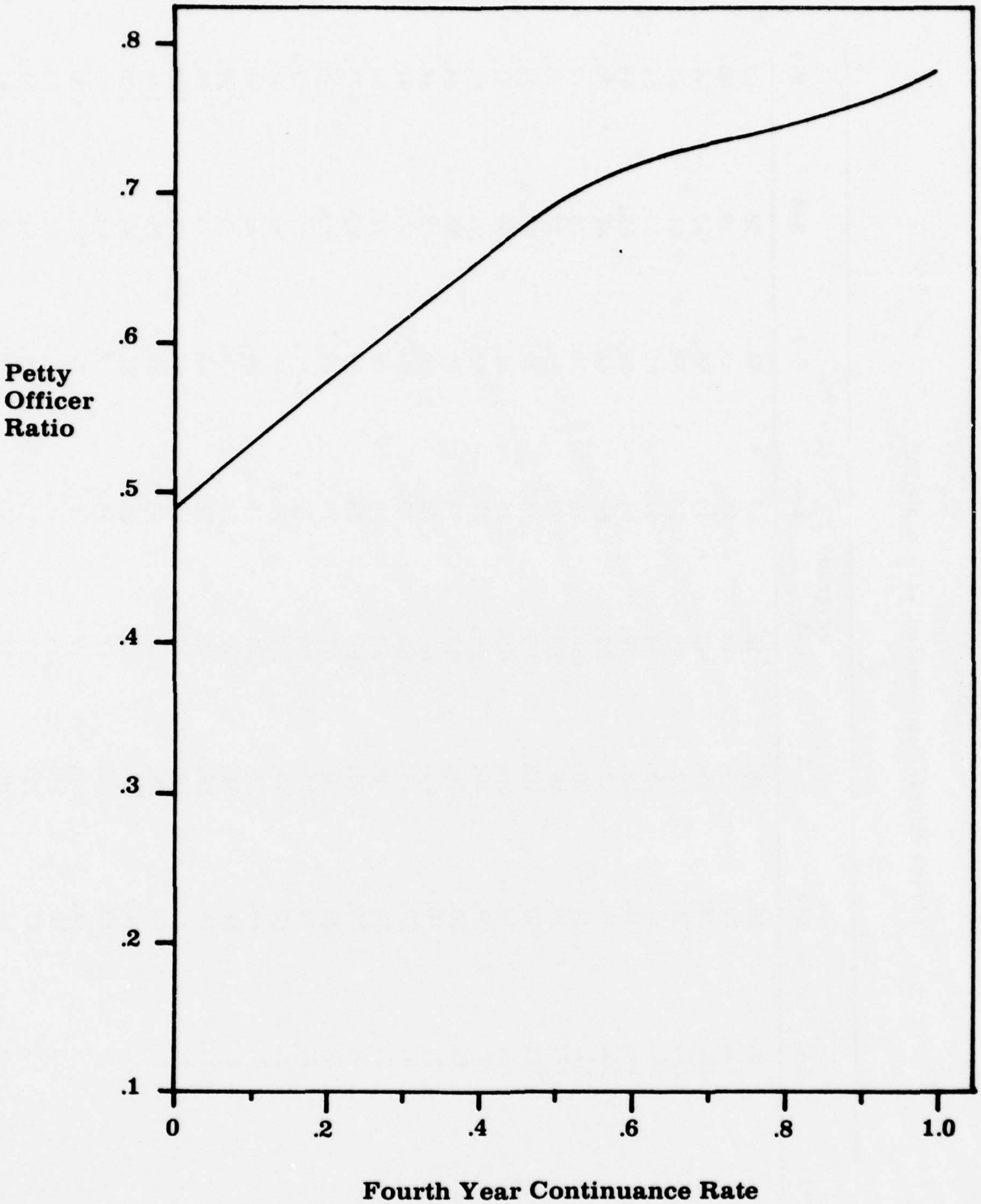

Figure 1. Petty officer ratio vs. continuance rate.

To find combinations of parameters compatible with the target ratio of 65 percent, the continuance rate runs were repeated for each of five advancement probabilities: 0.2, 0.4, 0.6, 0.8, and 1.0. In Figure 2, the petty officer ratio is plotted vs. the continuance rate for each advancement probability. Figure 3 shows the same data replotted for ease in interpolation. This format readily provides a number of discrete advancement probability/continuance rate combinations for any desired top six ratio. These combinations, plotted as shown in Figure 4, yield isoquants for a number of different petty officer ratios: each isoquant being the locus of compatible fourth-year continuance rates and advancement probabilities.

Given this range of alternatives, the next decision facing the planner is the identification of the most favorable feasible policy yielding a required petty officer ratio. This assessment can turn on a variety of considerations, including the proportion of careerists in the force (Figure 5), the number of nonprior service recruits each year (Figure 6), and preferences in regard to the age of the force (Figure 7), as well as questions of cost and utility.

For the purpose of evaluating the cost and utility impact of alternative policies, petty officer isoquants depicting the cost, utility, and cost per utile consequences of continuance and advancement policy combinations were also derived and are shown in Figures 8 through 10. Figures 11 and 12 display the average cost and cost per utile per man where the expected retirement costs of the 20- to 31-year career force have been included.

Some general observations concerning the costs and cost/benefits of these alternatives can be made. Given the simplifying assumptions inherent in this limited analysis, a cost comparison of TOP SIX policies can be made. For example, at the prevailing fourth-year continuance rate of 0.4, the cost to the Navy of maintaining a 65 percent TOP SIX ratio as opposed to a 60 percent ratio is approximately \$100.00 per man year. At any given petty officer ratio, it is clear that greater utility (at greater cost) is achieved by increasing continuance. The cost per utile presentation illustrates that the greatest gains are to be made by increasing continuance rates up to about 50 percent, with the point of diminishing returns being reached at higher rates.

It should be noted that the per capita cost model computes the personnel costs of the enlisted force as a function of the personnel continuance (reenlistment) and advancement patterns. It does not include the costs which the Navy may have to absorb to change prevailing patterns.

Within the optimization methodology, these system models are combined with a procedure for estimating proxy costs for changing the reenlistment behavior of the enlisted force. The resultant cost and cost per utile measures, including both the operating cost and the cost of achieving a more desirable operating point for the enlisted force, make up the objective function used in the optimization methodology.

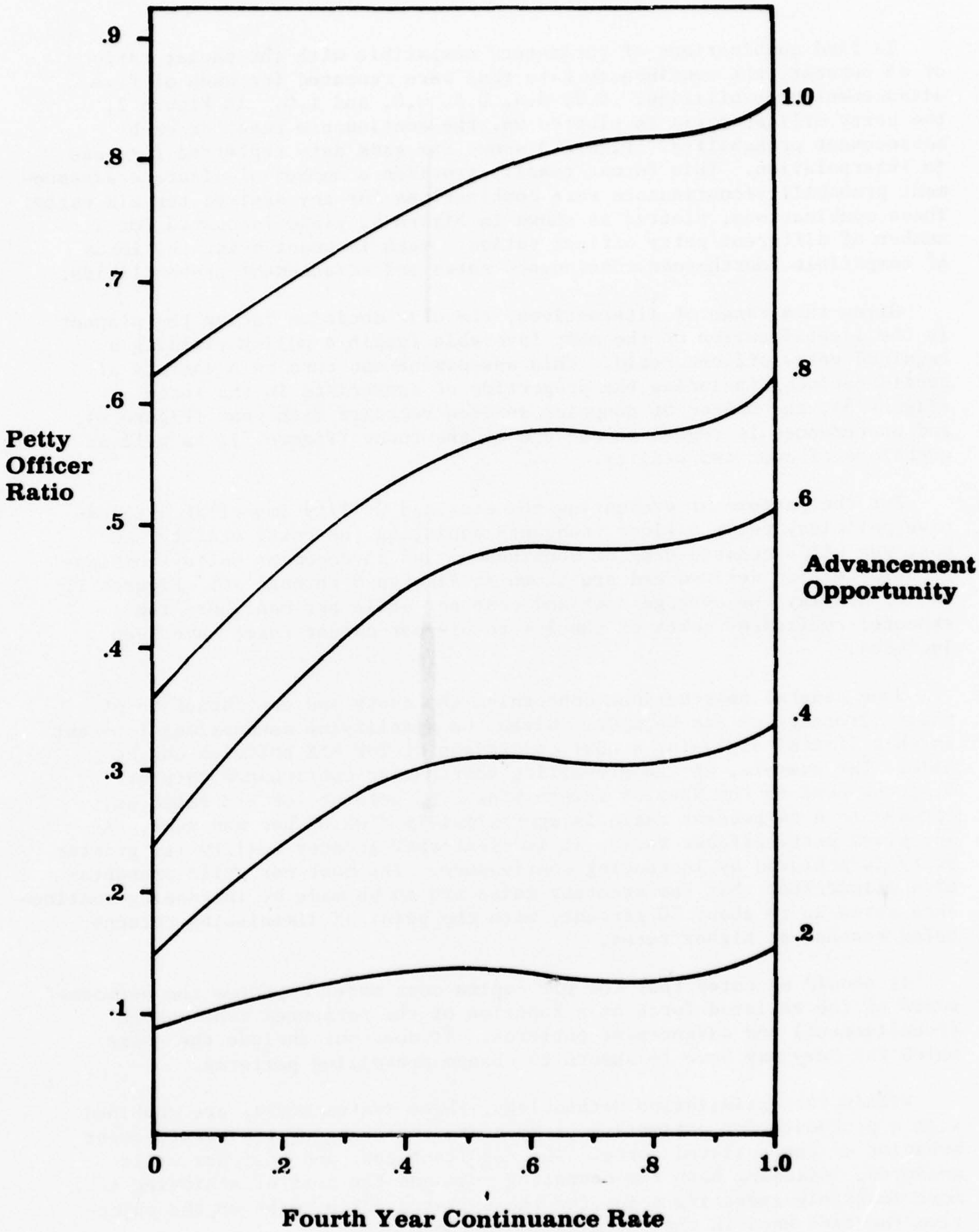

Figure 2. Petty officer ratio vs. continuance rate for each advancement opportunity (Interval .2)

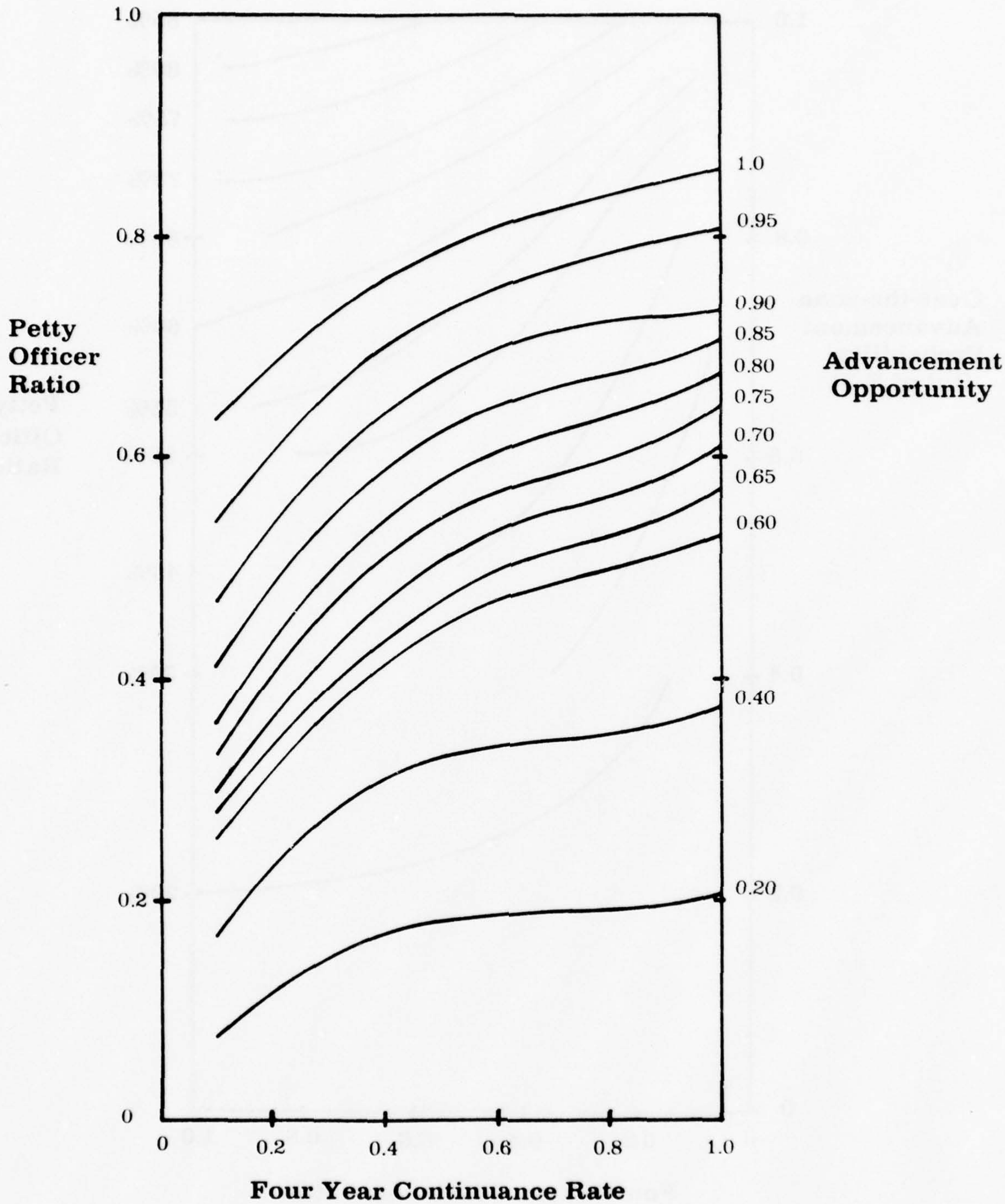

Figure 3. Petty officer ratio vs. continuance rate for each advancement opportunity (Interval .05).

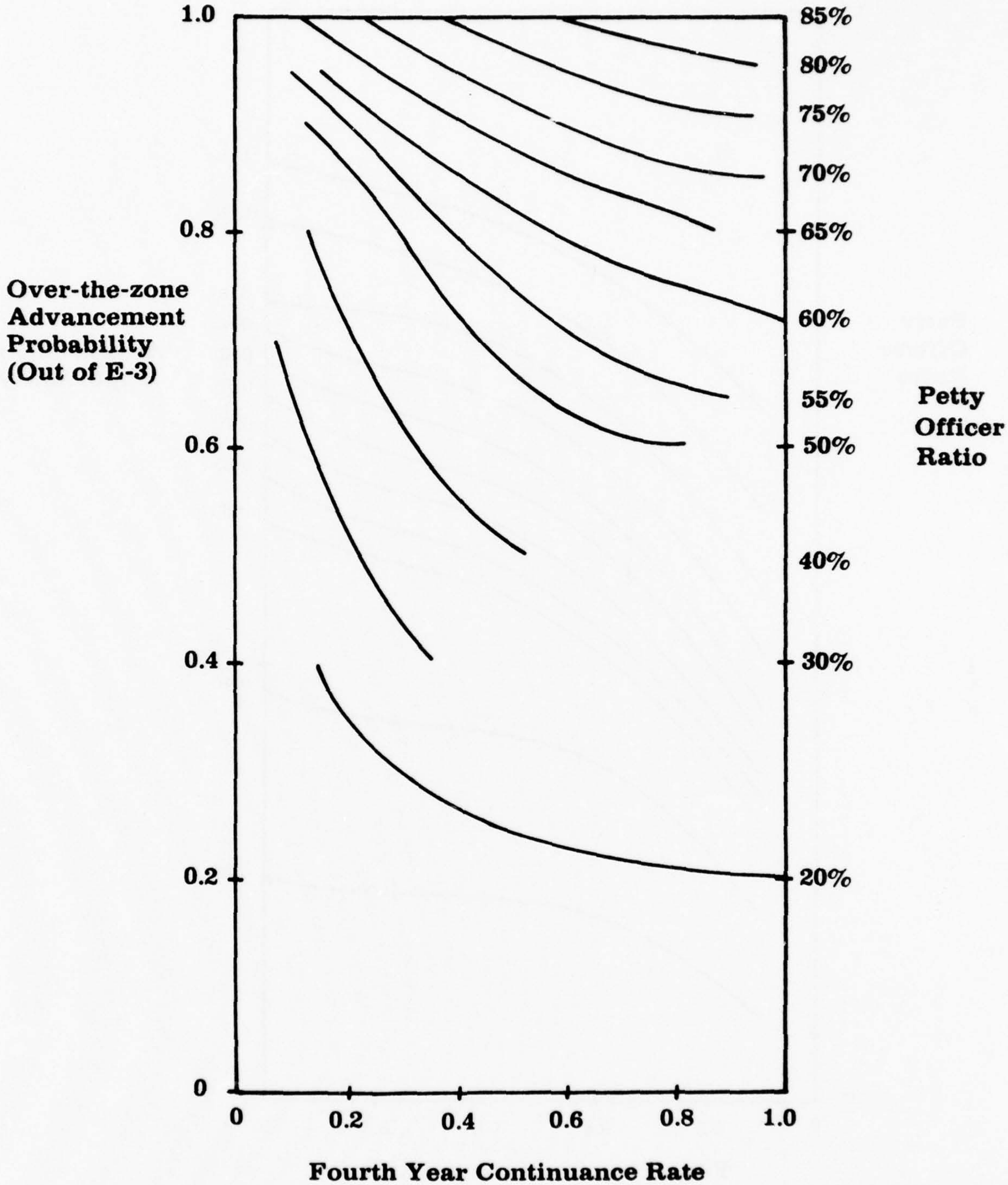

Figure 4. Advancement opportunity vs. continuance rate.

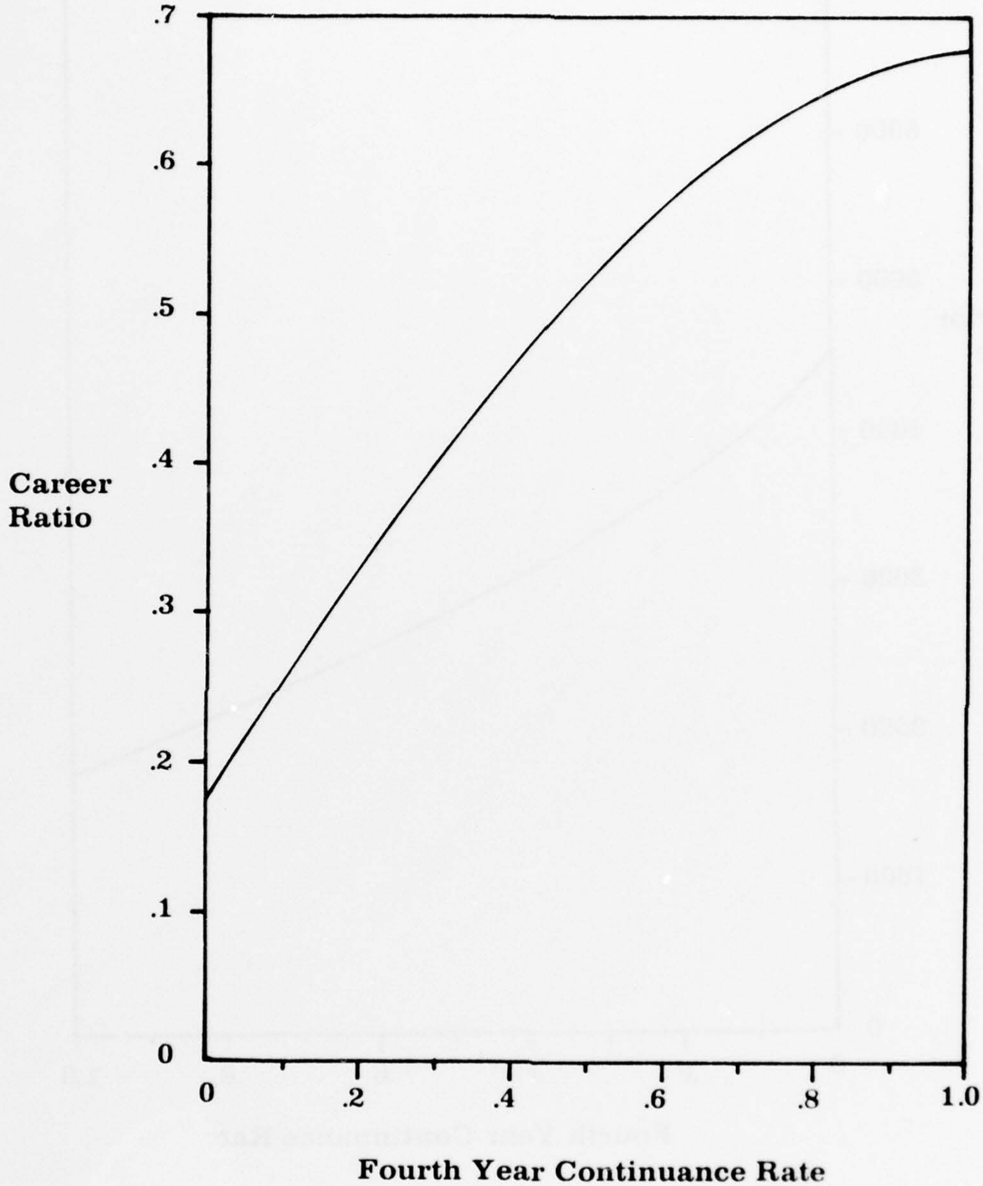

Figure 5. Career strength ratio vs. continuance rate.

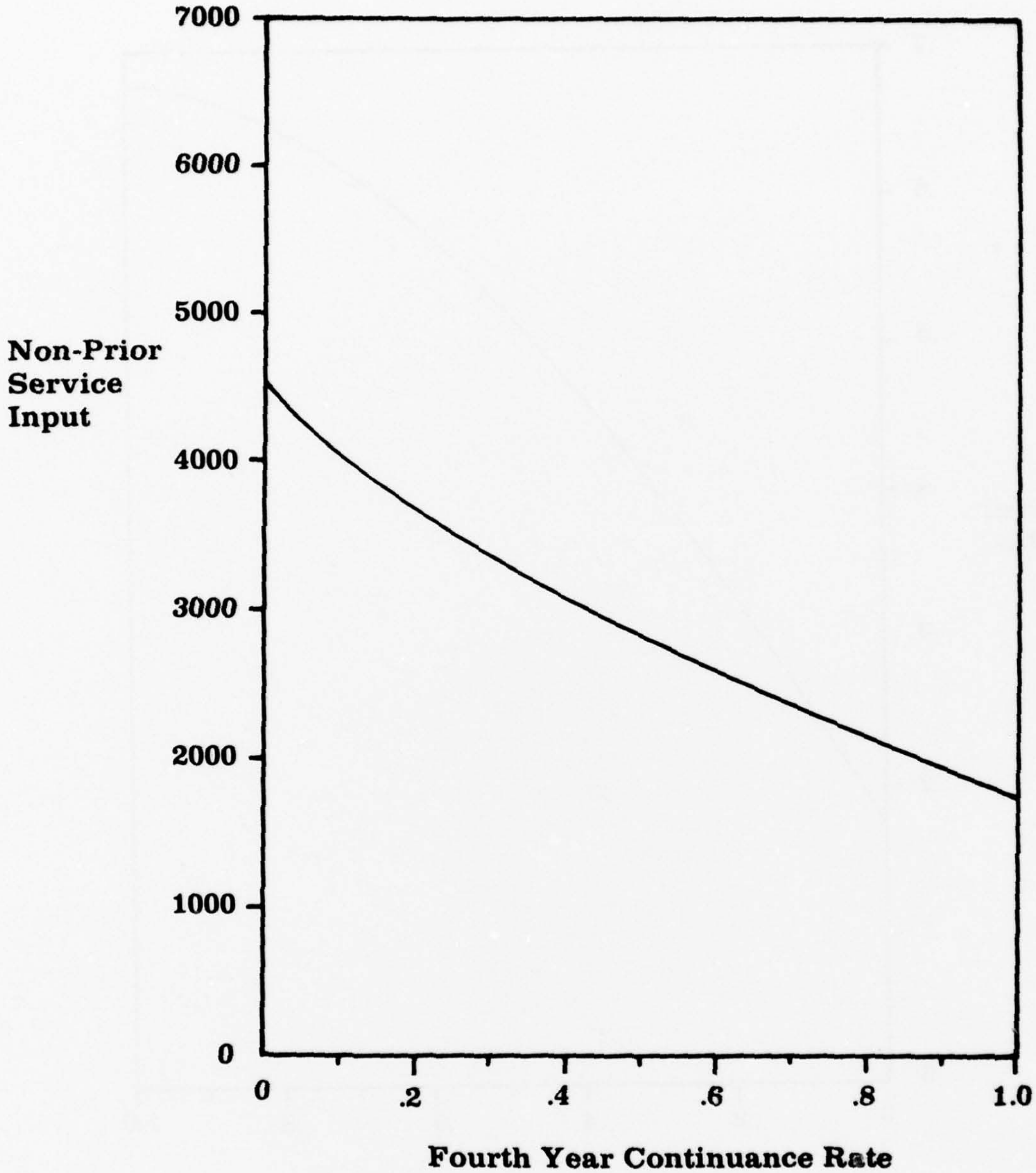

Figure 6. Nonprior service input required
vs. continuance rate.

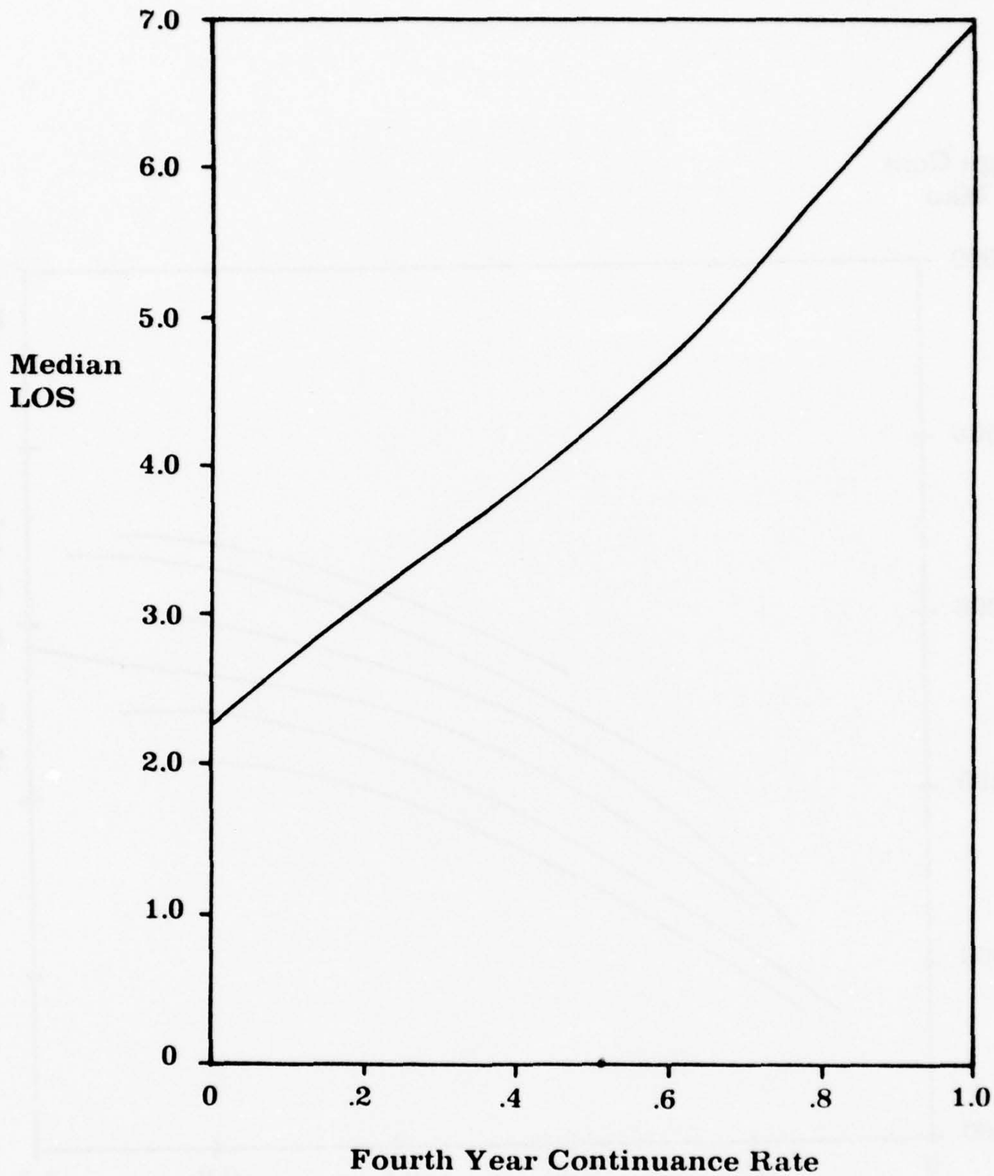

Figure 7. Median length of service vs. continuance rate.

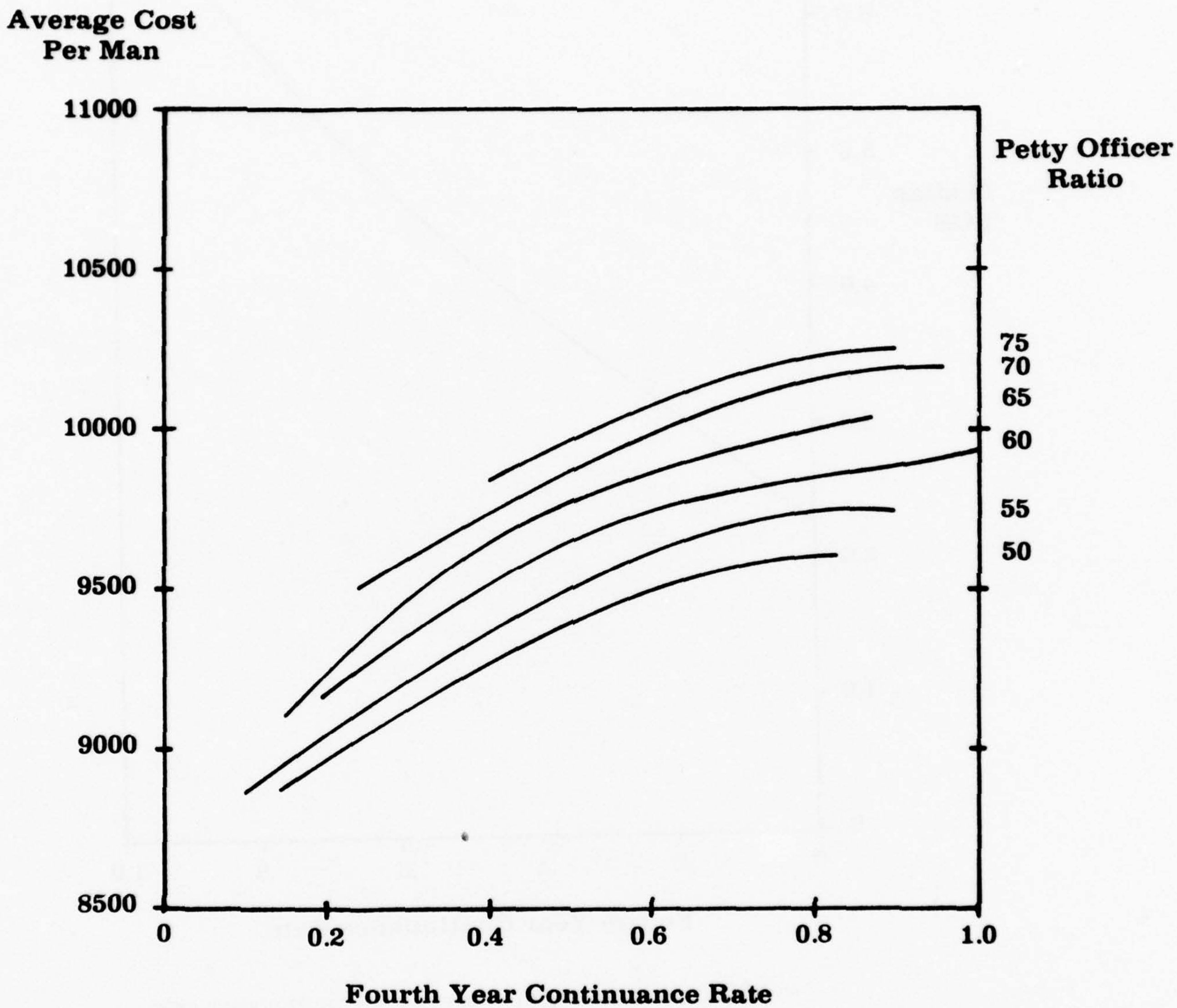

Figure 8. Average cost vs. continuance rate.

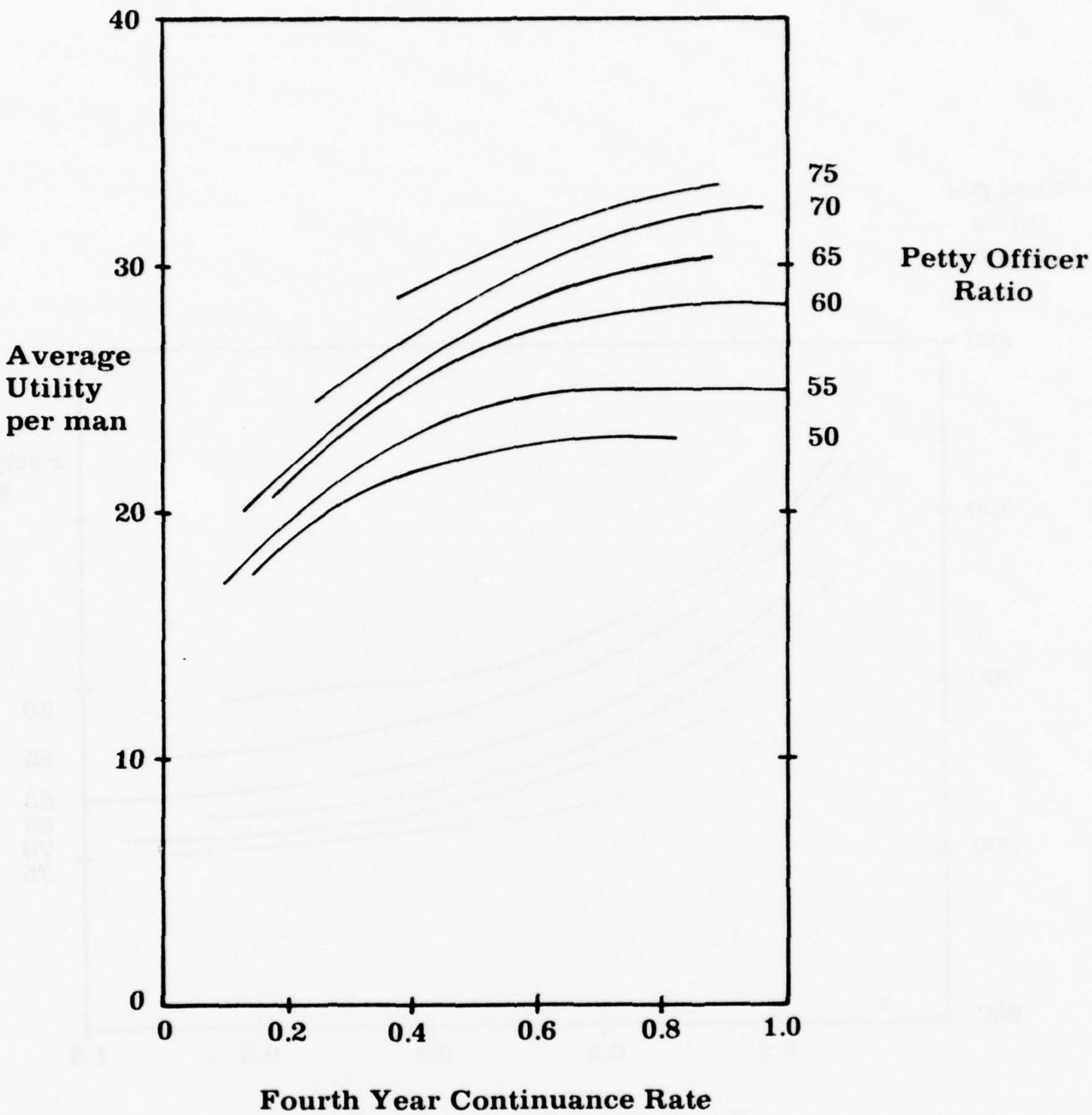

Figure 9. Average utility vs. continuance rate.

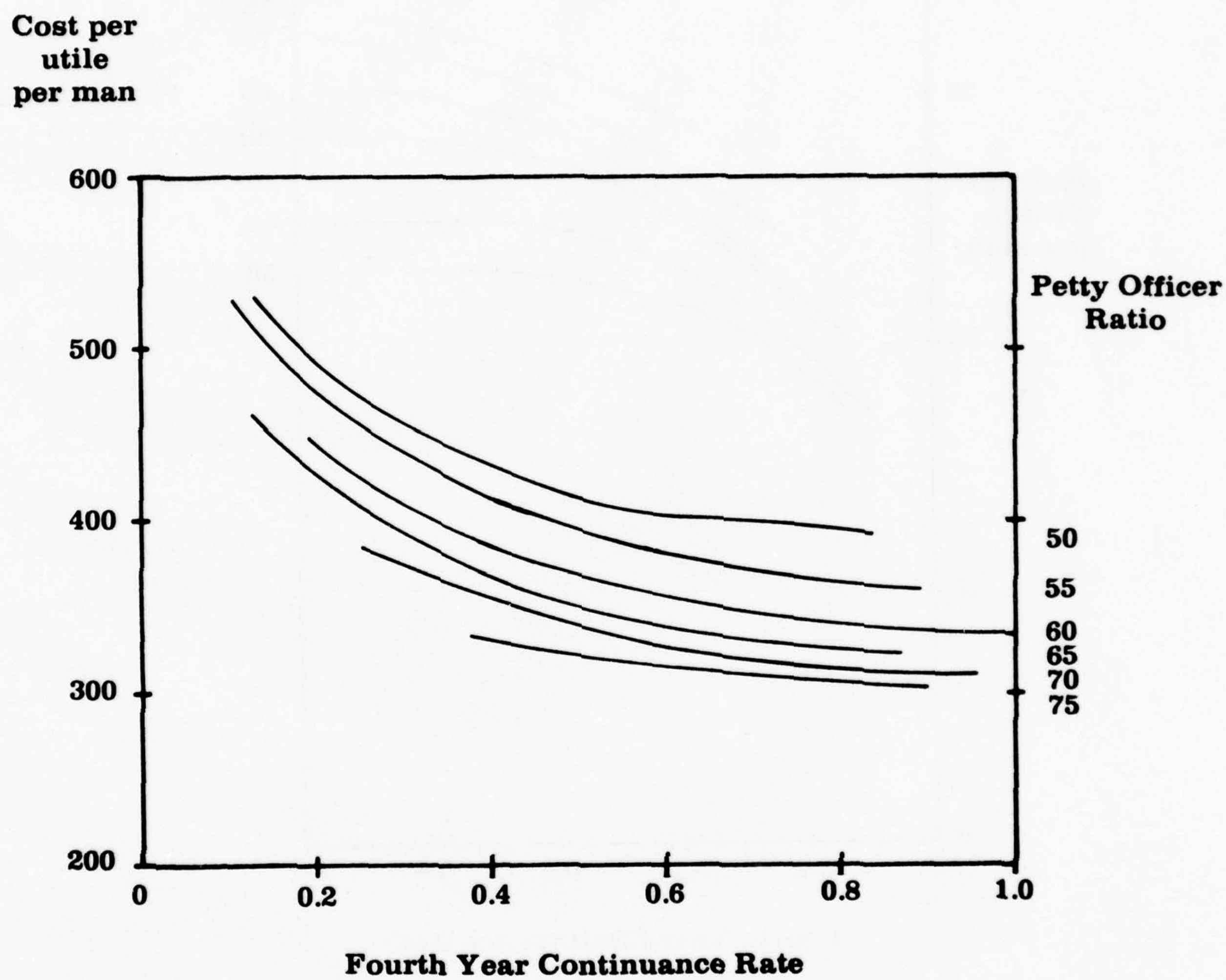

Figure 10. Cost per utile vs. continuance rate.

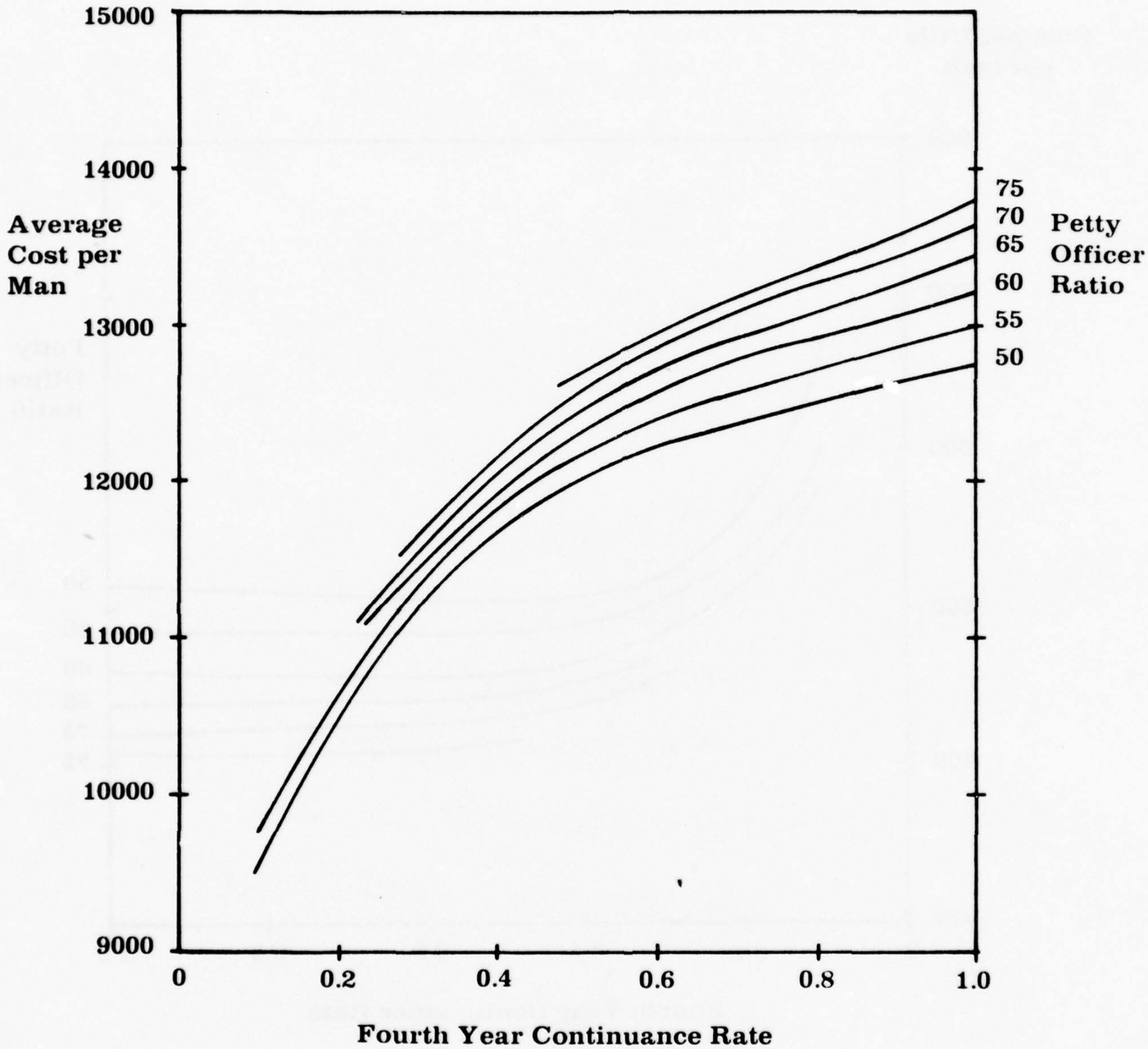

Figure 11. Average cost per man (including retirement) vs. continuance rate.

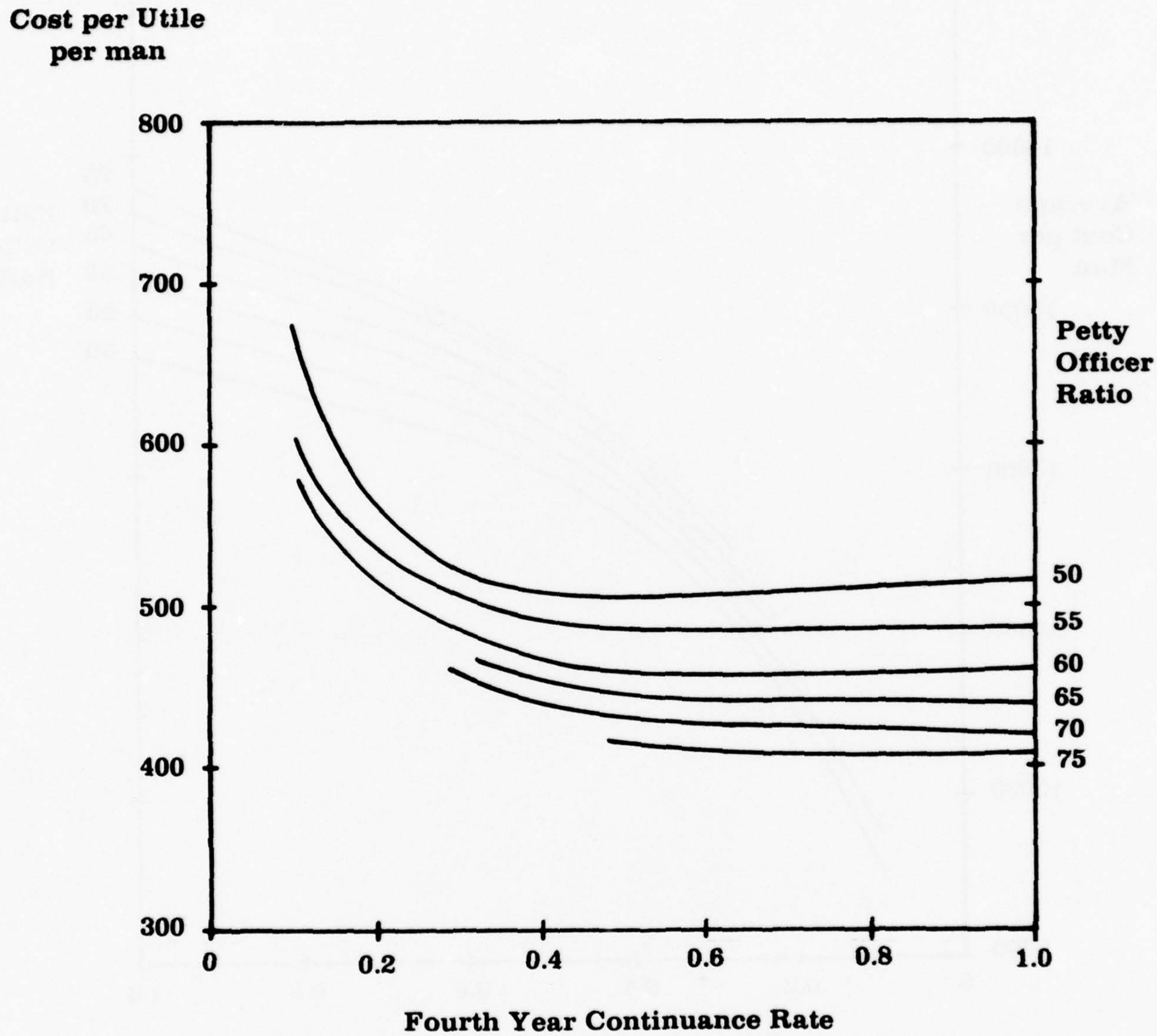

Figure 12. Cost per utile (including retirement) vs. continuance rate.

Reenlistment Elasticity Dependency Model

The previous section illustrated an application of the system models in concert for analyzing the sensitivity of personnel cost, inventory, utility, and other measures to changes in advancement and continuance parameters. Together, the models permit the planner to evaluate the cost/benefit impact of changes in Navy personnel policies on any rating or on ALNAV and, further, to perform systematic assessments of costs/benefits over a subset of personnel policies. The Elasticity Model was developed in recognition of the fact that a penalty cost is associated with changing the continuance behavior of the force. If one assumes an increase in the probability that enlisted personnel in a particular year-grade slot will continue in service, the supporting rationale must be taken into consideration; that is, one would expect that some action to increase reenlistments would be required--e.g., an increase in Variable Reenlistment Bonus. The converse can also be stated. If one assumes the probability of individuals in a particular year-grade slot that continue in service will decrease, again, there must be some supporting rationale; that is, there must be a penalty cost associated with this assumption (e.g., an increase in severance pay).

As a component of the optimization methodology, the inputs to the elasticity model are the desired reenlistment rates computed in the optimization model. The penalty cost estimates of the elasticity model are added to the total force costs to provide a realistic constraint on the range of solutions which may be identified as optimal.

Two methods of computing elasticity were used in the elasticity model. The provisions of the December 1971 Special Pays report issued by the Office of the Assistant Secretary of Defense (Manpower and Reserve Affairs) were incorporated in the computation of reenlistment bonus costs. The severance pay provisions in the Retirement Modernization Act were incorporated in the model to compute the costs of decreasing reenlistment or continuance.

Interpretation of Continuance Rates

The gain and loss behavior of the enlisted force is represented within the optimization model by an aggregate set of continuance rates for each rating. The rates define the probability that an individual will continue in the Navy from one year to the next and are used, together with other advancement rates and parameters, to compute the force distributions and per capita costs.

Reenlistments, along with all other gain/loss transactions, are not modeled as distinct personnel transactions in the model. It is in interpreting the impact of desirable changes in continuance behavior as proposed by the optimization model, in terms of the potential cost to the Navy of achieving that behavior, that the need for understanding the relationship between continuance and reenlistment arises. For the

purpose of introducing these potential costs into the model, data documented by DoD in the Special Pays report of 1971, describing the impact of various bonus levels on reenlistment behavior of the force, were selected. To make use of these data in the model, it was necessary to convert a proposed change in continuance behavior to a proposed change in reenlistment behavior.

Without getting into a detailed examination at this point, let it suffice to say that the methodology for definition, collection, aggregation, and reporting of separations, reenlistments (early, broken service, etc.), extensions, eligibility, and other related enlisted personnel transactions is quite complex and that the available options are numerous. Because of this and because of the aggregate level of detail of other components of the optimization methodology, a relatively simple conversion methodology was derived and implemented.

The reenlistment data source is the green book (BUPERS, NAVPERS 15658, FY 1973). The basic data elements involved in the optimization are:

1. First Term Reenlistment Rate (FTRR) by rating.

2. Career Reenlistment Rate (CRR) by rating.

3. First Term Eligibility Rate (FTRE), ALNAV.

4. Career Reenlistment Eligibility Rate (CARE), ALNAV.

For general ratings, the unadjusted reenlistment rates from the green book were used as reported. For expanded service ratings, the model inputs were derived from a weighted average (based on population) of the reenlistment rates of the compressed and service rating ratings.

In the source data, the reenlistment behavior of the enlisted man is differentiated by whether he is a first termer or a careerist. In contrast, reenlistment behavior in the model is represented as a function of length of service.

To distribute the first term and career source rates to the population by length of service, the following simplifying assumptions were made:

1. Stability in the behavior of the population with less than 1 year of service is assumed. This means that the continuance rates of the population with less than 1 year of service are held constant.

2. The reenlistment behavior of the population with 1 to 7 years of service can be approximated by the first-term reenlistment rate.

3. The reenlistment behavior of the population with 8 to 31 years of service can be approximated by the career reenlistment rate.

4. Within a specified length of service interval, the reenlistment rate of the population at each year of service is the same as the reenlistment rate of the total population in the interval.

5. Reenlistment eligibility rates, the ratio of the number of eligibles to the number of separations, are invariant by rating. ALNAV rates were attributed uniformly to each rating and were further attributed to the population at each LOS according to the above assumptions for attributing first-term and career-reenlistment behavior.

The basic relationship between reenlistment and continuance implemented in the model divides the population continuing from one year to the next into two subsets--those who have no choice (i.e., not involved in a separation action) and those who reach their end of active obligated service and are both eligible to and do reenlist.

The continuance rate formula is

CR_t	Those who continue
=	equal
$1 - REAOS_t$	those who have no choice
$+ REAOS_t \times RE_t \times RR_t$	plus those who reach EAOS and are eligible and reenlist.

where,

CR_t = number of people from LOS t who continue to LOS t + 1 in the following year divided by the total population at LOS t;

$REAOS_t$ = number of people at LOS t who reach end of active obligated service (EAOS) during year divided by total population at LOS t;

RE_t = number of eligibles at LOS t divided by the number of people reaching EAOS at LOS t; and

RR_t = number reenlisting at LOS t divided by the number both reaching EAOS and eligible at LOS t.

Given the current continuance rates for a given rating together with the basic reenlistment data, the model uses this relationship to compute REAOS for each LOS. In subsequent conversions of continuance rates to reenlistment rates, this initial set of REAOS is not changed, which amounts to the further assumption that the percentage of separation actions at a given LOS is invariant with change in continuance rate.

Each iteration through the optimization methodology involves an execution of the strength, cost, and utility models, followed by the conversion of the continuance rates to reenlistment rates and, finally, a computation of the reenlistment elasticity costs. After convergence to the "optimum" force distribution, the resultant reenlistment rates by LOS are aggregated into a first-term reenlistment rate and a career reenlistment rate.

Obviously, the above methodology does not attempt to model the multitude of transactions surrounding the reenlistment process. It has been implemented as an interim measure in need of further research prior to a wider application than it is now intended. Of course, this same statement can be made regarding the source data and assumptions upon which the elasticity costs are computed.

Despite the aggregate nature and inevitable bias of the conversion formulation, its use in the model is believed adequate. This is due to the fact that, in the elasticity costs, the differences between current and the proposed reenlistment rates are the dominant variables rather than the individual rates themselves.

Computation of Elasticity Costs

Table 5 depicts the reenlistment factors from the Special Pays Report. As shown, these factors estimate the change in first-term reenlistment (with no award) to be expected, given an additional VRB multiples--either 1, 2, 3, or 4. For example, if the prevailing reenlistment rate (with no award) is .40, then the payment of VRB multiples 1, 2, 3, or 4 is estimated to cause the reenlistment rate to rise to .42, .44, .48, or .50 respectively.

In the model, the assumption is made that the reenlistment contract length is 4 years. The computation of VRB pay shown below is in accordance with the present Navy policy:

VRB pay = VRB multiple x Reenlistment length
x monthly base pay.

The tables are transformed into elasticity curves (one for each reenlistment rate interval), which describe VRB costs in units of annual base pay as a function of reenlistment rate. Each curve is described by five points, the ordinates corresponding to VRB multiples 0, 1, 2, 3, and 4 or, equivalently, in terms of annual base pay units, 0, 1/3, 2/3, 3/3, and 4/3. The abscissas, respectively, are the free market reenlistment rate (or no award rate) and the estimate rates for each VRB multiple. To complete the curve to a reenlistment rate of 100 percent, an exponential curve was extrapolated through the data to provide a very large cost constraint for reenlistment rates exceeding the bounds of feasibility.

At LOS $\leq$ 4 years, these elasticity curves are used to compute the penalty costs of increasing reenlistment and the cost savings due to decreasing reenlistment. At lengths of service beyond 4 years, the elasticity curve applicable to LOS 4 is used as a proxy for estimating the penalty costs of increasing second and subsequent reenlistment rates.

Table 5

Factors for Estimating the Improved First Term Reenlistment Rate to be Attained from Award of Various Levels of Variable Reenlistment Bonus

First Term Reenlistment Rate (%) with No Award: More Than	and Not More Than	Multiply Reenlistment Rate Without Award by Variable Reenlistment Bonus[a]: V-1	V-2	V-3	V-4
0	10	1.30	1.35	1.65	1.75
10	15	1.30	1.35	1.65	1.70
15	20	1.25	1.30	1.55	1.60
20	25	1.20	1.25	1.45	1.50
25	30	1.15	1.20	1.35	1.40
30	35	1.10	1.15	1.25	1.30
35	50	1.05	1.10	1.20	1.25
50	--	1.00	1.05	1.10	1.15

[a]These factors were derived by averaging the estimated actual results for all services for the period FY 1966 - FY 1967 inclusive, interpolating between actual observations, and extrapolating outside actual observations.

To compute the expected costs of decreasing continuance of enlisted men with 5 or more years of service, the provisions of the Retirement Modernization Act regarding involuntary severance are used. Lump sum severance costs are computed as follows:

Severance costs = 10% of base pay (at time of severance) x LOS.

Optimization Methodology

Given the models discussed above, the optimization methodology requires one additional component--a procedure for exercising the models in tandem in a systematic search for the optimal set of variables which minimizes the objective function. An overview of the process may be described with reference to Figures 13 and 14. First, a set of input data reflecting the constraints imposed by the current environment is entered into the stable strength model (ASTATIC). ASTATIC calculates the resultant stable force which would result from these constraints and policies. This 9 x 31 inventory matrix represents what is feasible in terms of present continuation behavior and policy constraints. The resultant inventory is processed through the Per Capita Cost Model, which dynamically recalculates costs based on the inventory flows reflected in ASTATIC, and produces a 9 x 31 cost matrix representing the total cost of that particular force composition. At the same time, the inventory is multiplied by the 9 x 31 utility matrix to determine the total value of that force. From these two matrices, the cost per utile of that force is calculated and the initial conditions are set.

The optimizer then iteratively adjusts the continuance rates and advancement parameters singly or in combinations. After each adjustment, a new stable force composition is recalculated; the cost and utility models are recomputed; and the total cost, total utility, and cost per utile measures are calculated. When a change is made to a continuation rate, the penalty cost for affecting this change is calculated by the Elasticity Dependency Model and included in the cost computations. The system continues this iteration of changes in variables and sequential operation of models until a force composition resulting in a minimum cost per utile is obtained. This 9 x 31 matrix is the desired constrained optimum solution.

Important for an evaluation of the optimization are the assumptions, constraints, and determinants incorporated into the procedure, which in some fashion limit or determine the results obtained. Because the optimization process is at the heart of the Navy's program, these factors are discussed in some details here.

Assumptions

Ideally, advancements would be normally distributed over the permissable zone of advancement. Therefore, the model begins with a normally distributed advancement pattern and, during the process of optimization, the models are free to vary the mean of each paygrade distribution as necessary to reduce cost per utile.

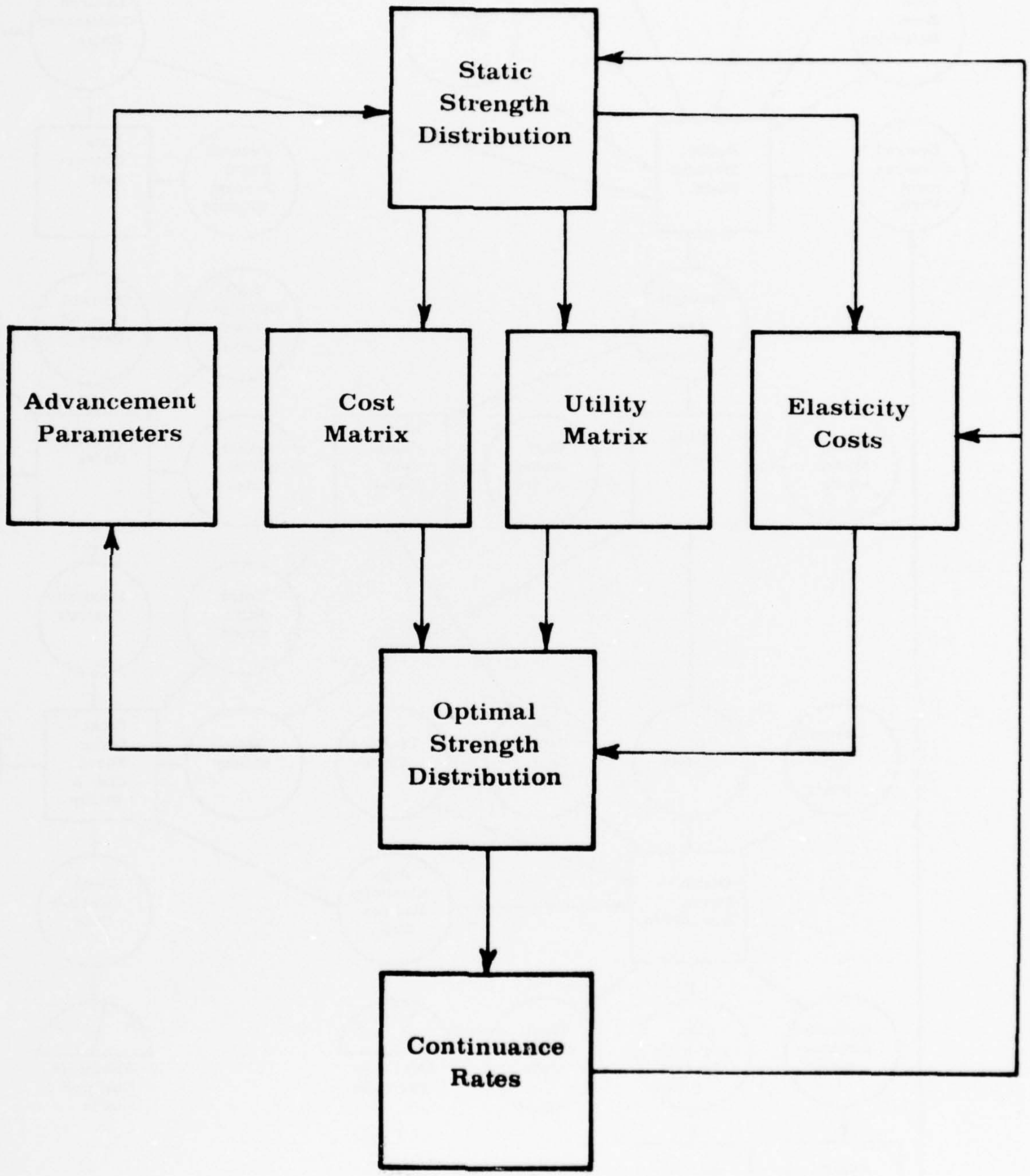

Figure 13. Optimization model overview.

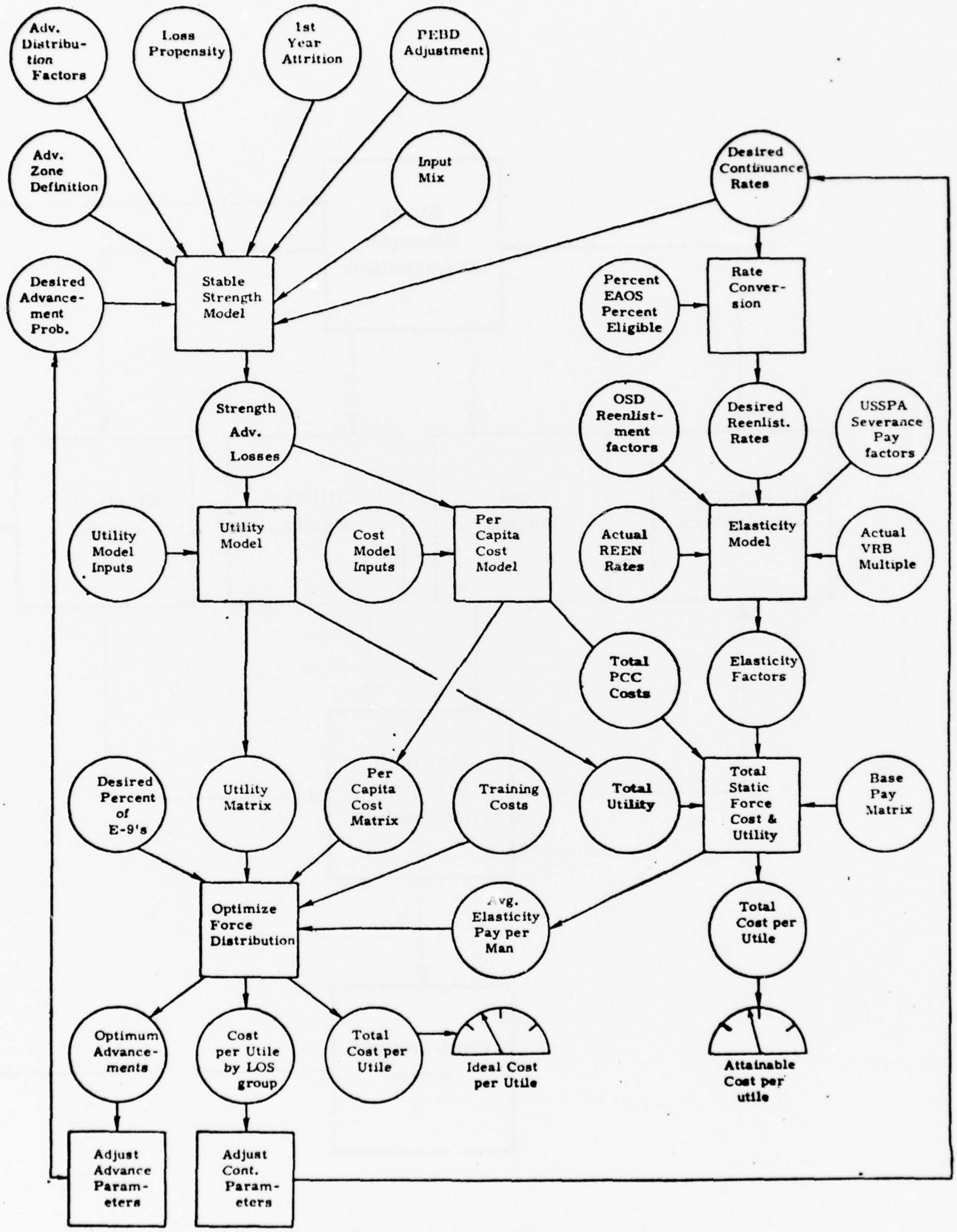

Figure 14. Optimization model block diagram.

The OSD Reenlistment Bonus Increment Table is a reasonable proxy for the penalty costs management must incur to increase continuation rates. The severance pay provision of the proposed DoD retirement pay legislation is a reasonable proxy for the penalty costs management incurs when decreasing continuation rates.

Constraints

The number of jobs to be done (interpreted as billets or bodies and operationalized as the total number of personnel in the matrix) is fixed. Regardless of the distribution by paygrade or LOS, the total number in the matrix must remain constant at the input value.

The number of E-9 billets specified by CNO for each skill is assumed valid. The model is constrained to provide at least the minimum required percentage of men in the E-9 paygrade.

Over-the-zone Advancement Opportunity is defined as the probability that a man will advance to the next higher paygrade, given that he has attained a given paygrade and not been lost to the Navy. The degree of selectivity required is a management judgment. It is input to the model as a constraint and is the same for every expanded service rating.

Advancement Zones were set at the low end by OSD constraints on time in grade and time in service and at the high end by existing or proposed Navy policy. The advancement zones are the same for each expanded service rating.

Determinants

A special class of inputs has been defined which essentially performs the function of orienting the optimization system to the current real world environment. These inputs are called determinants in that, to some extent, they predetermine the results the optimizer will obtain. That is, if different values of the determinant variables were to be inputted, the optimizer would arrive at a different optimum solution. They differ from constraints in that they do not predetermine the required values but, rather, place limits on the capability of the optimizer to make changes.

The determinants are (1) current continuance rates, (2) current CNO requirements adjusted to authorization levels (total and E-9 only for each expanded service rating), (3) current first-term and career reenlistment rates, (4) VRB level paid in FY73 (the last year included in the derivation of the continuance rates), (5) OSD Reenlistment Bonus Increment Tables, and (6) the severance pay levels from current DoD proposed retirement legislation.

The above paragraphs give a broad overview of the process and some detail about the inputs. The following traces the optimization subsystem through the cycle in more detail.

The number of jobs-to-be-done is taken directly from the CNO requirements, as adjusted to an authorization of 61.5 percent petty officer ratio and a 475,000 man end-strength. For a given rating, these CNO

authorizations are further adjusted by attribution of nondesignated strikers and allocation of billets from general and compressed ratings, as necessary, to form an expanded service rating. Aside from these details, it is important to note that the number of positions to be filled by skill is accepted as an externally imposed constraint. The possibility that an optimum force may be a more efficient force and, therefore, may reduce the total number of men required is not addressed in the current methodology.

Continuance rates were derived for both the total Navy and each expanded service rating by a process of reverse weighting the last 9 years of historical continuance rates. Since nondesignated strikers had not been attributed in the historical data, continuance rates could not be calculated for the first enlistment years of service in each service rating. For this purpose, All-Navy continuance rates for initial enlistment cohorts by length of initial contract were used for the first three LOS cells; service rating continuance rates specify the character of the stable force along the LOS vector, assuming current policies continue into the future.

Initial advancement parameters were set as follows. Using All-Navy continuance rates, derived as described above, and authorized paygrade levels in a 475,000 man force, a series of iterations was performed searching out feasible sets of advancement opportunities and advancement distributions. From among the feasible sets, a selection was made by the Navy based on providing acceptable levels of selection through the advancement system. Advancement opportunity is expressed in terms of Over-the-Zone Advancement Opportunity, the probability of a man at a given paygrade advancing to the next higher grade given that he remains in the Navy.

As a function of the selected Over-the-Zone Advancement Opportunities, the stable force model (ASTATIC) was exercised for each expanded service rating by using individual rating continuance rates, rating jobs to be done and paygrade authorizations to determine a feasible set of advancement distributions. This determined the mean and standard deviation of the advancement distribution for each paygrade in each expanded service rating. The combination of these factors then provided a feasible set of advancement parameters for entry into the optimizer. It should be noted that this step does not simply replicate paygrade authorization. Given that the observed historical continuance rates prevail, it is possible to have paygrade structures that cannot be supported. ASTATIC comes as close as possible to the authorization levels by paygrade.

The lower limits of the advancement zones were set by OSD policy limitations for minimum time in service at advancement. It was unnecessary to incorporate the "waiverable to" provision of OSD policy except at paygrade E-5 where an additional year of time in service eligibility was required. No attempt was made to limit the population in this extra year to any fixed proportion of the paygrade population.

The initial conditions specified for the optimizer, then, describe what is feasible under current constraints. For each expanded rating, the current state was set by inputting current continuance rates, number of

jobs to be done, percent of service rating required to be E-9, and a feasible set of advancement parameters. It should again be emphasized that the individual paygrade authorizations, except for E-9, were not constraints but rather products of the feasible sets of inputs. Every effort was made, however, through initial specification of the Over-the-Zone Advancement Opportunity, to develop a feasible set as close as possible to the current 61.5 percent overall Navy Top Six ratio.

The optimizer then takes this initial feasible stable condition and adjusts continuation rates and advancement parameters in the search for an optimum. Continuance rates are aggregated into management groups and the group continuance rates are the variables optimized. The groups are LOS 1, LOS 2-3, LOS 4, LOS 5-7, LOS 8-10, LOS 11-20, and LOS 21 and above. LOS cell 1 continuance is not variable; therefore, the optimizer is manipulating six continuance rate variables representing the aggregate group continuance. The advancement parameter, which the optimizer is free to adjust, is the mean of the distribution of advancement probabilities, given the high and low year constraints on the zone of advancement and the desired Over-the-Zone Advancement Opportunity described above.

The optimization model iteratively varies these parameters by means of a variable metric optimization algorithm (Davidson-Fletcher-Powell) (Mayberry, 1973, Note 1) designed to converge on the optimum through a steepest descent technique. The iterations of the models continue until no further decrease in cost per utile is observed. By definition, the resultant stable force composition is an optimum one. (See appendix for discussion of Variable Metric Techniques.)

As discussed above, the dependent variable of optimization, cost per utile, is a function of 12 variables, six continuance rates, and six others related to the advancement policy (each variable marks the location of the mean of the advancement distribution). Within the cost and strength models, these variables can only take on values over a specific range. For example, a continuance rate is only defined for values between 0 and 1, since it quantifies the probability that a man with a given number of years of service will continue in the Navy for another year. The mean of each advancement distribution, that is, the LOS in which the greatest numbers of advancements out of the associated paygrades are made, must lie within the zone of advancement.

There is no intrinsic mechanism in the function minimization algorithm to constrain changes made to any of these variables to these acceptable ranges. For this purpose, each continuance rate (CR) was subjected to the following transformation:

$$CR_i = \frac{1}{2} + \frac{1}{2} \tanh (X_i)$$

The function minimization algorithm varies the variables X_i as it explores the range of the function, with X_i free to take on any value between $-\infty$ and $+\infty$. A similar transformation is made to the advancement means to constrain these variables to lie in the zone of advancement.

Example

The process can best be illustrated and understood by a review of the output of various stages of the optimization. The following figures represent this output. It should be recognized that the data used here is for purposes of illustration. The definitive data on optimum forces by rating will be presented in the "white book" report (Department of the Navy, FY 1975).

Table 6, the Requirements Audit Trail, shows the changes in the paygrade structure at each stage of the process. The audit trail shows the conversion of CNO requirements to authorization levels, the attribution of nonrated billets to the rating, and the allocation of general and compressed rating billets (resulting in the paygrade structure of the expanded service rating). From this initial set of authorizations, a feasible set of advancement policies for the particular rating is developed. The paygrade composition of this feasible inventory illustrates how closely authorizations can be approached by the personnel inventory. The paygrade vector resulting from optimization is included and can be compared to the feasible to show the changes between current and optimum continuance and advancement policies.

Table 6

Requirements Audit Trail

Item	E1-E3	E-4	E-5	E-6	E-7	E-8	E-9
CNO Requirement (2/28/74)	550	1,100	1,140	940	535		
CNO Requirements (61.5 Top Six 475K Stable Force)		1,031	1,047	860	527		
Attributed Base	1,996						
High Paygrade Allocation						139	48
Expanded Service Rating	1,996	1,031	1,047	860	527	139	48
Feasible Inventory	2,000	1,023	1,042	905	486	144	48
Optimum Inventory	1,974	1,333	783	856	487	137	48

Table 7 is a matrix solution from the stable force model which shows the BM rating composition that would result from the continuance of current continuation rates and the advancement policies as described above. Table 8 shows the same force after optimization. To highlight the differences, summary statistics are contained in Table 9. In this case, the optimal force composition proposes a considerable increase in first-term reenlistment and a considerable decrease in career reenlistments. The total severance cost and added reenlistment payments illustrate the penalty costs which the optimizer was willing to pay in order to obtain the optimum distribution.

As shown, cost per utile of the optimum force composition is almost ten dollars less than the initial feasible force composition. By multiplying this difference by the total utility of the optimum force, an efficiency index, measured in dollars, is obtained. Since the efficiency index includes the penalty costs as well as the per capita costs, it is a cost-benefit measure of the gains potentially available in adopting optimum advancement and continuance policies.

Table 7

Feasible Inventory Distribution for BM Rating

LOS	E1	E2	E3	E4	E5	E6	E7	E8	E9	Total
1	0.0	0.0	2306.00	0.0	0.0	0.0	0.0	0.0	0.0	2306.00
2	0.0	0.0	2605.00	0.0	0.0	0.0	0.0	0.0	0.0	2605.00
3	0.0	0.0	248.68	2164.32	0.0	0.0	0.0	0.0	0.0	2413.00
4	0.0	0.0	185.46	104.49	1731.05	0.0	0.0	0.0	0.0	2021.00
5	0.0	0.0	52.40	27.34	491.26	0.0	0.0	0.0	0.0	571.00
6	0.0	0.0	34.35	17.92	107.68	216.04	0.0	0.0	0.0	376.00
7	0.0	0.0	33.35	17.52	79.95	238.18	0.0	0.0	0.0	369.00
8	0.0	0.0	30.70	16.61	76.55	229.15	0.0	0.0	0.0	353.00
9	0.0	0.0	23.26	14.53	69.97	211.24	0.0	0.0	0.0	319.00
10	0.0	0.0	16.18	13.84	67.46	200.52	0.0	0.0	0.0	298.00
11	0.0	0.0	10.44	11.85	60.33	65.38	120.00	0.0	0.0	268.00
12	0.0	0.0	8.04	10.16	58.46	45.93	132.40	0.0	0.0	255.00
13	0.0	0.0	6.21	9.62	56.84	44.55	127.78	0.0	0.0	245.00
14	0.0	0.0	4.33	9.00	55.43	43.82	124.41	0.0	0.0	237.00
15	0.0	0.0	3.02	8.43	54.14	43.22	94.87	27.32	0.0	231.00
16	0.0	0.0	2.60	8.16	53.22	42.85	70.55	50.61	0.0	228.00
17	0.0	0.0	2.05	7.46	50.91	41.92	54.19	51.94	12.53	221.00
18	0.0	0.0	1.84	7.34	49.80	41.79	46.31	44.82	72.10	219.00
19	0.0	0.0	1.52	6.85	48.42	41.04	42.27	36.58	37.31	214.00
20	0.0	0.0	0.76	4.60	39.31	33.96	34.64	26.90	35.82	176.00
21	0.0	0.0	0.08	0.77	21.48	21.13	21.37	13.69	23.48	102.00
22	0.0	0.0	0.03	0.28	13.99	14.98	14.50	8.66	15.57	68.00
23	0.0	0.0	0.02	0.16	9.39	10.88	9.73	5.76	10.08	46.00
24	0.0	0.0	0.01	0.10	7.39	5.55	7.98	4.53	8.44	34.00
25	0.0	0.0	0.01	0.04	5.31	4.42	6.36	3.74	7.12	27.00
26	0.0	0.0	0.00	0.04	3.69	3.28	4.93	3.08	5.98	21.00
27	0.0	0.0	0.00	0.03.	2.98	2.65	3.98	2.54	4.82	17.00
28	0.0	0.0	0.00	0.03	2.73	2.46	2.89	2.38	4.51	16.00
29	0.0	0.0	0.00	0.02	2.22	2.02	2.51	2.13	4.09	13.00
30	0.0	0.0	0.00	0.02	1.84	1.77	2.51	1.53	3.63	11.00
31	0.0	0.0	0.00	0.02	1.43	1.26	1.42	0.88	2.00	7.00
32	0.0	0.0	5576.28	2461.56	3223.23	1609.98	925.31	287.08	202.46	14285.89
PERCENTAGE			39.0334	17.2307	22.5623	11.2697	6.4771	2.0096	1.4172	
CAREERISTS			231.	193.	1492.	1610.	925.	287.	202.	4941.
RETIREMENTS			2.	7.	48.	41.	41.	31.	44.	214.
TIS ADVANCEMENT			2.0026	3.0013	5.1103	10.1382	15.2286	17.2953	0.0	
TIS SERVING			1.4506	3.1380	6.3925	10.1010	14.5826	17.9794	20.6155	
ANNUAL ADV. OPP.			0.4337	0.8560	0.1399	0.0920	0.0900	0.1746	0.0	
ZONE ADV. OPP.			0.9000	0.9500	0.7501	0.7500	0.6500	0.7000		
ZONE			2.-8.	3.-10.	5.-16.	10.-22.	14.-26.	16.-28.		

NON PRIOR SERVICE INPUT: 3117 148. 2 YEAR; 0. 3 YEAR; 2970. 4 YEAR 0. 6 YEAR

Table 8

Optimum Inventory Distribution for BM Rating

LOS	E1	E2	E3	E4	E5	E6	E7	E8	E9	Total
1	840.26	999.08	375.66	0.0	0.0	0.0	0.0	0.0	0.0	2215.00
2	135.76	1072.77	1291.47	0.0	0.0	0.0	0.0	0.0	0.0	2500.00
3	10.07	33.80	191.60	2085.53	0.0	0.0	0.0	0.0	0.0	2321.00
4	2.46	15.02	161.34	95.06	1675.11	0.0	0.0	0.0	0.0	1949.00
5	0.84	3.50	58.78	32.14	592.74	0.0	0.0	0.0	0.0	688.00
6	0.0	0.84	40.57	21.09	119.15	273.35	0.0	0.0	0.0	455.00
7	0.0	0.0	40.41	20.69	96.98	289.92	0.0	0.0	0.0	448.00
8	0.0	0.0	37.27	19.64	93.19	278.91	0.0	0.0	0.0	429.00
9	0.0	0.0	28.30	17.20	85.23	257.26	0.0	0.0	0.0	388.00
10	0.0	0.0	19.19	16.33	82.01	243.47	0.0	0.0	0.0	361.00
11	0.0	0.0	12.23	13.94	73.15	213.77	10.91	0.0	0.0	324.00
12	0.0	0.0	5.36	8.94	67.44	103.44	108.82	0.0	0.0	294.00
13	0.0	0.0	2.34	7.70	62.60	52.88	143.48	0.0	0.0	269.00
14	0.0	0.0	0.89	6.18	57.85	48.14	133.93	0.0	0.0	247.00
15	0.0	0.0	0.28	4.76	52.95	45.73	103.43	21.84	0.0	229.00
16	0.0	0.0	0.07	3.92	48.00	43.59	76.49	42.93	0.0	215.00
17	0.0	0.0	0.04	2.94	41.95	40.85	55.79	42.23	14.19	198.00
18	0.0	0.0	0.02	2.56	34.64	39.82	44.87	37.61	26.58	186.00
19	0.0	0.0	0.01	2.02	31.57	37.53	38.34	32.14	32.39	174.00
20	0.0	0.0	0.00	1.14	23.77	29.04	29.11	24.14	27.80	135.00
21	0.0	0.0	0.00	0.11	12.05	17.10	16.88	11.97	16.90	75.00
22	0.0	0.0	0.00	0.04	7.79	12.07	11.37	7.69	11.03	50.00
23	0.0	0.0	0.00	0.02	5.09	8.57	7.41	5.01	6.90	33.00
24	0.0	0.0	0.00	0.01	3.98	4.28	6.05	3.92	5.75	24.00
25	0.0	0.0	0.00	0.01	2.82	3.38	4.77	3.22	4.81	19.00
26	0.0	0.0	0.00	0.00	1.99	2.54	3.74	2.67	4.07	15.00
27	0.0	0.0	0.00	0.00	1.58	2.02	2.98	2.18	3.24	12.00
28	0.0	0.0	0.00	0.00	1.49	1.92	2.41	2.08	3.09	11.00
29	0.0	0.0	0.00	0.00	1.10	1.45	1.97	1.78	2.69	9.00
30	0.0	0.0	0.00	0.00	0.97	1.32	1.80	1.42	2.48	8.00
31	0.0	0.0	0.00	0.00	0.75	0.93	1.15	0.81	1.35	5.00
32	0.0	0.0	5380.21	2361.98	3277.92	2053.28	805.72	243.64	163.17	14285.90

UNADJUSTED MATRIX = 2632.89 TARGET = 14286.00

	E3	E4	E5	E6	E7	E8	E9	Total
PERCENTAGE	37.66	16.53	22.95	14.37	5.64	1.71	1.14	
CAREERISTS	251.	181.	1603.	2053.	806.	244.	163.	5301.
RETIREMENTS	0.	2.	32.	38.	37.	30.	36.	174.
TIS ADVANCEMENT (OUT)	2.00	3.00	5.07	11.25	15.43	17.02	0.0	
TIS SERVING (IN GRADE)	1.45	3.01	6.07	9.72	14.94	18.05	20.20	
ANNUAL ADV. OPP.	0.43	0.86	0.16	0.08	0.10	0.18	0.0	
ZONE ADV. OPP.	0.90	0.95	0.75	0.75	0.65	0.70		
ZONE	2 - 8	3 - 10	5 - 16	10 - 22	14 - 26	16 - 28		

NON PRIOR SERVICE INPUT: 2991 142. 2 YEAR: 0. 3 YEAR: 2850. 4 YEAR 0. 6 YEAR

Table 9

Evaluation of BM Rating Inventory

Item	Feasible	Optimum
First Term Reenlistment	12.1%	22.4%
Career Reenlistment	91.8%	74.9%
LOS 4 Continuance	.28	.35
Non-Prior-Service Input	3117	2991
Percent E-9	1.42%	1.14%
Top Six	60.9%	52.3%
Additional Reenlistment Bonus	---	3.543M
Additional Severance Pay	---	1.024M
Total Cost	190.211M	188.745M
Total Utility	576,893	589,881
Cost Per Utile	329.7	320.0
Cost Per Utile Increment	---	9.7
Cost Benefit Index[a]	---	5.721M

[a] Defined as change in cost per utile between rating feasible and optimum distributions multiplied by total number of units of utility in optimum force. Can be viewed as a dollar measurement of improved efficiency.

FINDINGS

Exercise of the optimization methodology for each of 87 Navy ratings has been completed. Table 10 displays several cost/benefit measures for each rating. The first column lists the efficiency index computed for each rating. This measure summed over all ratings indicates a $97.6M cost-benefit advantage of the ALNAV optimum force distribution over the feasible force distribution. It may also be noted that this cost-benefit advantage includes the additional VRB and severance costs of $88.5M projected by the elasticity model as necessary to shape the feasible force inventories into correspondence with the optimum force inventory.

Tables 11 and 12 display the ALNAV feasible and "optimum" inventory distributions. These aggregate inventories were obtained by summing the optimum inventory distributions over all ratings.

An insight into the optimization model results can be had by examining these two inventory distributions. For example, the feasible inventory distribution illustrates that, over the long term (steady state), the observed continuance behavior of the enlisted force will support only a 33.73 percent career ratio. The optimizer, by adjusting the continuance rates for each rating, increased the steady state career ratio to 35.66 percent. By way of comparison, the actual ALNAV career ratio is approximately 43 percent. Other significant differences between the optimized inventory and the feasible inventory include:

1. Increase in LOS 4 continuance rate from .35 to .38.

2. Decrease in LOS 10 through 19 continuance rates, each by .01 or .02.

3. Increase in TOP SIX ratio from 59.44 percent to 61.07 percent.

4. A younger enlisted inventory, and particularly a younger careerist inventory.

5. A greater number of E-5s, E-6s, and E-7s.

6. An increase in average paygrade serving from 3.990 to 4.056.

7. A decrease in number of retirees from 6131 to 6009, annually.

Table 10

Cost Benefit Measure by Rating

Rating	EFF Index ($)	Added VRB ($)	Severance Cost ($)	Total Cost ($M) Feasible	Total Cost ($M) Optimum	Difference ($M) Opt. - Feas.
ABE	36,727	363,307	938	21.3	21.9	0.6
ABF	404,226	635,953		19.6	20.7	1.1
ABH	980,982	1,150,470	80,513	38.6	40.4	1.8
AC	557,168	1,017,683	3,561	52.6	54.3	1.7
ADJ	3,613,042	3,616,409	223,971	171.1	178.6	7.5
ADR	743,257	787,412	13,129	45.4	46.7	1.3
AE	1,524,533	1,841,195	293,738	141.2	141.7	0.5
AG	278,723	130,802	1,579	29.9	30.1	0.2
AK	2,890,526	991,194	519,876	64.6	62.0	-2.6
AME	1,025,939	764,546	40,049	42.9	44.5	1.6
AMH	2,991,131	2,331,642	456,070	99.5	100.6	1.1
AMS	3,015,542	2,098,594	397,767	120.5	121.8	1.3
AO	773,072	1,129,546	79,208	91.8	93.4	1.6
AQ	1,327,086	1,267,174	90,973	74.2	77.0	2.8
ASE	216,998	364,755		9.7	10.3	0.6
ASH	272,344	267,037	9,543	9.6	10.1	0.5
ASM	336,306	207,254	26,732	17.4	17.6	0.2
AT	4,384,233	2,405,211	256,813	206.2	209.4	3.2
AW	430,890	463,246	51,243	47.5	47.7	0.2
AX	404,020	-140,661	8,966	26.6	25.9	-0.7
AZ	2,168,167	1,178,224	175,156	57.5	59.5	2.0
BM	5,721,845	3,542,818	1,023,850	190.2	188.7	-1.5
BT	832,984	1,688,544	90,307	181.2	183.8	2.6
BU	1,012,211	579,580	143,676	42.5	52.7	0.2
CE	605,115	239,390	88,673	25.8	25.7	-0.1
CM	453,633	449,375	23,037	23.5	24.4	0.9
CS	4,912,940	1,858,607	343,053	175.2	178.7	3.5
CTA	122,233	144,927	2,995	13.7	13.9	0.2
CTI	232,461	542,054	1,128	18.2	19.1	0.9
CTM	318,510	207	10,411	34.7	34.6	-0.1

Table 10 (Continued)

Rating	EFF Index ($)	Added VRB ($)	Severance Cost ($)	Total Cost ($M) Feasible	Optimum	Difference ($M) Opt. - Feas.
CTO	551,896	757,994	34,696	31.1	32.2	1.1
CTR	736,540	1,007,822	54,892	37.8	39.2	2.6
CTT	531,282	1,206,078	303	33.8	36.0	2.2
DK	483,584	318,401	61,434	35.8	36.2	0.4
DM		158,030	212	5.9	6.1	0.2
DP	399,373	695,378	47,967	48.3	49.5	0.2
DS	368,458	-128,459	19,240	33.3	32.9	-1.6
DT	1,527,937	1,471,890	143,836	47.3	49.0	1.7
EA	82,826	49,613	370	5.0	5.1	0.1
EM	1,452,666	2,190,019	159,290	235.0	238.4	3.4
EN	916,891	1,300,558	195,842	127.3	128.2	0.9
EO	739,639	956,373	43,381	35.2	37.0	1.8
ETN	2,665,487	0	22	175.0	176.1	1.1
ETR	2,053,311	0	334	180.4	181.0	0.6
EW	685,796	-300,677	43,795	40.8	40.5	0.3
FTB	348,632	56,810		16.7	16.9	0.2
FTG	866,142	940,982	208,191	78.3	78.3	0
FTM	416,452	58,969	63,304	68.0	67.8	-0.2
GMG	1,383,572	1,936,298	107,313	79.3	82.7	3.4
GMM	272,116	2,181		22.8	22.9	0.1
GMT	362,990	416,903	27,128	34.4	35.0	0.6
HM	4,203,222	5,138,927	501,673	311.5	318.0	6.5
HT	1,835,329	2,100,302	265,076	206.6	209.4	2.8
IC	576,555	1,137,417	70,785	104.5	106.2	1.7
IM	27,710	2,327		8.1	8.1	0
JO	127,133	81,231	1,953	10.2	10.4	0.2
LI	160,466	171,231	0	7.1	7.5	0.4
ML	45,563	19,465	0	3.4	3.4	0
MM	6,990,817	-1,250,929	1,055,363	436.5	426.9	-9.5
MN	117,627	353,133		10.7	11.2	0.5
MR	162,284	180,234	31,414	42.9	43.0	0.1

Table 10 (Continued)

Rating	EFF Index ($)	Added VRB ($)	Severance Cost ($)	Total Cost ($M) Feasible	Optimum	Difference ($M) Opt. - Feas.
MT	295,194	-203,178		31.7	31.2	0.5
MU	369,071	536,696	35,247	19.9	20.8	0.9
OM	62,997	225,304	1,218	6.0	6.2	0.2
OS	1,677,550	2,009,051	124,120	109.2	111.8	2.6
OT	226,977	54,059	1,630	24.9	24.8	-0.1
PC	617,990	436,612		19.7	20.8	1.1
PH	281,710	745,131	90,620	37.4	37.9	0.5
PM	73,478	90,430		2.3	2.5	0.2
PN	1,323,041	1,677,027	165,157	102.8	105.5	3.3
PR	668,542	605,503	123,973	30.9	30.9	0
PT	172,020	312,209		10.1	10.6	0.5
QM	294,152	631,252	49,039	67.5	68.4	0.9
RM	2,385,695	4,730,132	274,392	316.1	323.0	6.9
SD	1,226,992	115,518	29,422	91.5	91.9	0.4
SH	2,527,801	1,576,989	191,756	83.3	86.7	3.4
SK	4,007,227	2,538,436	699,788	163.4	163.3	0.1
SM	423,287	784,958	156,200	52.8	52.7	0.1
STG	980,344	1,441,719	39,008	75.3	77.6	2.3
STS	655,946	88,868	53,412	45.3	45.5	0.2
SW	584,263	328,180	79,650	15.0	15.2	0.2
TD	142,117	707,907	17,615	27.7	28.8	1.1
TM	1,819,730	1,992,804	351,415	90.6	91.1	0.5
UT	688,683	395,400	93,689	23.1	23.2	0.1
YN	2,298,207	3,172,134	230,430	193.6	200.1	6.5
TOTALS	97.6M	78.0M	10.4M	6245.7M	6331.4M	87.3M

Table 11

Feasible ALNAV Inventory

Year	E-1	E-2	E-3	E-4	E-5	E-6	E-7	E-8	E-9	Yr Totals
1	27980.	33269.	12509.	.	.	.	.	.	.	73759
2	4949.	39111.	47084.	.	.	.	.	.	.	91143.
3	590.	1979.	11219.	72667.	.	.	.	.	.	86454.
4	92.	561.	6021.	12225.	44657.	.	.	.	.	63555.
5	31.	129.	2167.	3264.	16404.	.	.	.	.	21995.
6	.	29.	1409.	1853.	5338.	6469.	.	.	.	15121.
7	.	.	1038.	1178.	3748.	7114.	.	.	.	13078.
8	.	.	793.	820.	3131.	6833.	.	.	.	11578.
9	.	.	534.	584.	2609.	6346.	.	.	.	10073.
10	.	.	314.	479.	2345.	5887.	.	.	.	9025.
11	.	.	223.	385.	2061.	3110.	2408.	.	.	8188.
12	.	.	176.	317.	1943.	2167.	3166.	.	.	7769.
13	.	.	140.	297.	1857.	1865.	3289.	.	.	7449.
14	.	.	99.	272.	1783.	1696.	3308.	.	.	7159.
15	.	.	75.	250.	1714.	1573.	2587.	733.	.	6932.
16	.	.	60.	230.	1627.	1473.	2170.	1148.	.	6707.
17	.	.	52.	211.	1547.	1390.	1909.	1184.	205.	6497.
18	.	.	42.	201.	1443.	1335.	1727.	1167.	407.	6321.
19	.	.	35.	184.	1389.	1270.	1571.	1117.	565.	6130.
20	.	.	19.	127.	1132.	1031.	1252.	911.	581.	5053.
21	.	.	3.	38.	687.	680.	805.	538.	443.	3194.
22	.	.	1.	18.	466.	491.	548.	359.	331.	2213.
23	.	.	1.	11.	325.	361.	371.	244.	239.	1552.
24	.	.	.	8.	252.	177.	289.	186.	208.	1120.
25	.	.	.	3.	169.	135.	215.	146.	179.	849.
26	.	.	.	3.	126.	105.	169.	121.	158.	681.
27	.	.	.	2.	93.	78.	127.	92.	124.	516.
28	.	.	.	2.	76.	66.	66.	77.	109.	396.
29	.	.	.	1.	54.	48.	52.	63.	95.	314.
30	.	.	.	1.	39.	38.	42.	39.	77.	236.
31	.	.	.	1.	29.	26.	27.	22.	46.	151.
GRADE TOTALS:										
	33642.	75077.	84016.	95630.	97044.	51788.	26098.	8145.	3766.	475198.

	E-4	E-5	E-6	E-7	E-8	E-9	Yr Totals
PERCENTAGE	20.12	20.42	10.90	5.49	1.71	0.79	
TOP SIX PERCENT = 59.44							
CAREERISTS	10739.	52387.42	51788.	26098.	8145.	3766.	160293.
OVER 4 YEARS = 33.73% OR 160293.							
RETIREMENTS	184.	1389.	1229.	1405.	1050.	839.	6131.
TIS ADVANCEMENT (IN)	2.07	3.14	5.48	10.96	15.73	18.05	
TIS ADVANCEMENT (OUT)	3.14	5.48	10.96	15.73	18.05	0.0	
TIS SERVING (IN GRADE)	3.24	6.64	10.28	15.16	18.44	21.19	
ANNUAL ADV. OPP.	0.72	0.15	0.09	0.08	0.12	0.0	
ZONE ADV. OPP	0.97	0.79	0.76	0.67	0.72		

Table 12

Optimized ALNAV Inventory

Year	E-1	E-2	E-3	E-4	E-5	E-6	E-7	E-8	E-9	Yr Totals
1	27168.	32304.	12146.	.	.	.	.	.	.	71619.
2	4805.	37968.	45709.	.	.	.	.	.	.	88480.
3	477.	1600.	9071.	72764.	.	.	.	.	.	83911.
4	84.	515.	5526.	8657.	46947.	.	.	.	.	61729.
5	31.	131.	2192.	2839.	18558.	.	.	.	.	23751.
6	.	30.	1456.	1637	5784.	7387.	.	.	.	16294.
7	.	.	1125.	1083.	4135.	7906.	.	.	.	14248.
8	.	.	888.	785.	3409.	7645.	.	.	.	12727.
9	.	.	604.	578.	2866.	7043.	.	.	.	11090.
10	.	.	356.	485.	2595.	6507.	.	.	.	9943.
11	.	.	253.	397.	2287.	3217.	2871.	.	.	9024.
12	.	.	181.	311.	2142.	2295.	3510.	.	.	8439.
13	.	.	127.	285.	2028.	1933.	3593.	.	.	7966.
14	.	.	79.	252.	1922.	1746.	3541.	.	.	7540.
15	.	.	50.	221.	1817.	1612.	2835.	649.	.	7184.
16	.	.	36.	195.	1692.	1503.	2310.	1119.	.	6856.
17	.	.	29.	172.	1574.	1411.	1940.	1172.	245.	6543.
18	.	.	22.	160.	1412.	1355.	1737.	1134.	457.	6276.
19	.	.	17.	142.	1337.	1281.	1567.	1068.	597.	6009.
20	.	.	9.	94.	1064.	1020.	1224.	861.	579.	4853.
21	.	.	1.	29.	641.	670.	780.	511.	426.	3058.
22	.	.	1.	13.	435.	485.	528.	345.	314.	2120.
23	.	.	.	8.	301.	356.	353.	235.	222.	1476.
24	.	.	.	6.	235.	178.	276.	182.	193.	1070.
25	.	.	.	3.	157.	136.	205.	145.	164.	810.
26	.	.	.	2.	116.	106.	160.	121.	144.	647.
27	.	.	.	1.	85.	76.	118.	92.	111.	485.
28	.	.	.	1.	70.	65.	64.	78.	98.	376.
29	.	.	.	1.	50.	47.	50.	63.	85.	297.
30	.	.	.	1.	36.	37.	40.	39.	68.	220.
31	.	.	.	.	26.	25.	25.	23.	41.	141.
GRADE TOTALS:										
	32566.	72547.	79878.	91121.	103719.	56043.	27727.	7838.	3743.	475173.
PERCENTAGE				19.18	21.83	11.79	5.84	1.65	0.79	
TOP SIX PERCENT = 61.07										
CAREERISTS				9700.	56772.	56043.	27727.	7838.	3743.	169441.
OVER 4 YEARS = 35.66% OR 169440.										
RETIREMENTS				142.	1337.	1249.	1405.	1034.	824.	6009.
TIS ADVANCEMENT (IN)				2.04	3.10	5.43	10.80	15.75	17.79	
TIS ADVANCEMENT (OUT)				3.10	5.43	10.80	15.75	17.79	0.0	
TIS SERVING (IN GRADE)				3.17	6.54	10.09	14.95	18.48	20.94	
ANNUAL ADV. OPP.				0.75	0.15	0.09	0.08	0.13	0.0	
ZONE ADV. OPP.				0.97	0.78	0.75	0.66	0.71		

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CONCLUSIONS

First, the objective function minimizes cost per utile. In doing so, it allows the amount of cost and utility to vary. Although results showed the optimum force to have a higher utility than the feasible force, the optimum force cost over $87M more.

In minimizing cost per utile, the decision maker has no control over the absolute amount of either cost or utility. However, in real life decision making, the planner is usually under a fixed budget constraint in the short term or striving for a desired utility in the long term. Under the minimum cost per utile formulation, neither short- or long-term objectives are being met.

Second, the model optimizes each skill rating separately. This produces different ratios of cost to utility in each skill rating and, consequently, a suboptimal solution for the whole Navy. The optimal solution for the whole Navy would be when the ratios of marginal cost to marginal utility are equal for every rating. However, the model for all skill ratings taken together is too large to solve computationally on a computer. The suboptimal solution obtained is the result.

Finally, a computer-based optimization methodology has been developed to help determine optimum force distributions by skill grouping, length of service, and paygrade. The initial results of the model have been positively accepted at the Bureau of Naval Personnel. The model is being implemented as a personnel planning and management tool at BUPERS.

RECOMMENDATIONS

It is recommended that other objective functions besides the present one of minimizing cost per utile be investigated. One alternative objective function could maximize the utility of the enlisted force subject to a constant budget. A second alternative objective function could minimize the cost of the enlisted force subject to a utility constraint.

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APPENDIX

A VARIABLE METRIC TECHNIQUE
APPLIED TO ENLISTED FORCE OPTIMIZATION

A VARIABLE METRIC TECHNIQUE APPLIED TO ENLISTED FORCE OPTIMIZATION

Introduction

The basic criterion to be optimized for the enlisted force is to minimize cost per utile. The cost of a man-year in a specific rating, specific length of service (LOS), and specific grade may be estimated fairly readily (although only those who have attempted to perform the details of this estimation will be aware of the many subtle alternatives which must be confronted before useful numerical results can be obtained). The utility data, however, has been much more difficult; the estimates of utility have been distilled from the judgment of experienced Navy managers by a variety of methods, including the Delphi technique and other analyses, which are reported elsewhere (Schmid & Hovey, 1976).

The procedure for finding the optimum feasible ratio of cost to utility is not straightforward, because it is not possible to put precise limits on the set of feasible policies. (Certain ranges of such key variables as reenlistment rates are regarded as "normal," and extreme variations from current experience will be viewed with great skepticism, but the limits of normality cannot be definitely established.)

What has been done instead is to examine steady-state solutions generated by a computer program ASTATIC which employs overall retention rates and promotion opportunities to generate a plausible distribution of the EMs in a single rating; the input parameters to ASTATIC are then varied in such a way as to match the distribution, as closely as possible, to the so-called "ideal" distributions that are found to be optimal when fewer constraints are considered. (Calculation of those "ideal" distributions uses Lagrange Dynamic Programming, as described below.) The output from ASTATIC, representing a distribution believed to be feasible, is used to modify those components of cost estimates and utility estimates which are sensitive to the whole pattern of retention and promotions. Cost of changing retention rates is also included; these costs may be in the form of Variable Reenlistment Bonus (VRB) payments to increase retention or payment of severance pay and vested pension if retention must be decreased. The Davidon-Fletcher-Powell (DFP) method is used to vary the group reenlistment rates so as to achieve this minimum cost per utile in the realistic force distribution. That procedure is also described below. The zone promotion opportunities (i.e., the fraction of entrants to a grade who will eventually be promoted to the next higher grade) are also needed as inputs to ASTATIC, but are estimated from the "ideal" distribution rather than being active search parameters in the DFP algorithm.

Lagrange Dynamic Programming

The Lagrange Dynamic Programming (LDP) algorithm is used to optimize the promotion rates in a rating, given the retention rates, costs, and utility data. The index i denotes LOS throughout, j denotes pay grade, r_i is the retention rate for EM's with LOS = i, c_{ij} is the cost per man-year, and u_{ij} is the utility per man-year for EMs with LOS = i and grade = j.

We then attempt to minimize

$$\frac{\sum_{ij} c_{ij} x_{ij}}{\sum_{ij} u_{ij} x_{ij}}$$

where the x_{ij} are feasible population values. If there were no other constraints, we would attempt to choose a promotion-pattern (i.e., select the x_{ij}) such that we find

$$\min (\lambda) \text{ such that } \lambda \geq \frac{\sum cx}{\sum ux} \text{ has a solution for some } x,$$

$$\text{i.e., } \min (\lambda) \text{ such that } \frac{\lambda \sum ux - \sum cx}{\sum ux} \geq 0 \text{ has a solution;}$$

$$\text{i.e., } \min (\lambda) \text{ such that } \sum x_{ij}(\lambda u_{ij} - c_{ij}) \geq 0 \text{ has a solution;} \qquad (1)$$

$$\text{i.e., } \min (\lambda) \text{ such that } \max_x \sum x_{ij}(\lambda u_{ij} - c_{ij}) \geq 0;$$

$$\text{i.e., } (\lambda) \text{ such that } \quad \max_x \sum x_{ij}(\lambda u_{ij} - c_{ij}) = 0.$$

We solve these equations by finding successively a trial value λ_o for λ, solve $\max_x \sum x_{ij}(\lambda_o u_{ij} - c_{ij})$ by standard DP or optimal-flow methods,

replace λ_o by $\lambda = \dfrac{\sum c_{ij} x_{ij}}{\sum u_{ij} x_{ij}}$,

and iterating until convergence occurs.

We actually wish to impose the requirement that at least 1 percent of the force is at grade E-9; this means that $\dfrac{\sum_i x_{i9}}{\sum_{ij} x_{ij}}$ must be $\geq p_g = 0.01$,

$$\text{i.e., that } \sum_{ij} x_{ij} \alpha_j \geq 0 \text{ where } \alpha_j = \frac{1-p_g \text{ for } j=9}{-p_g \text{ for } j\neq 9} . \qquad (2)$$

If the previous solution to condition (1) satisfies (2), we are done; otherwise, we must find $\mu > 0$ such that

$$\max_x \sum x_{ij}(\lambda u_{ij} - c_{ij} + \mu\alpha_j) = 0 \text{ and } \sum x_{ij}\alpha_j = 0, \tag{3}$$

because (3) follows from (1) and (2).

The determination of the appropriate value of μ is not as efficient as the above procedure for λ, but must be a "cut-and-try" procedure.

This completes the description of the LDP portion of the algorithm, except to note that we actually consider j as a continuous variable representing average grade at LOS = i.

DFP Optimization

Fletcher and Powell (1963) present a method for minimizing a function $f(y_1,\ldots\ldots,y_n)$ of several variables $y_1,\ldots.,y_n$. Their method, which we call the "DFP" algorithm (for Davidon-Fletcher-Powell), belongs to the class of "variable-metric" methods described in Davidon's (1969) classical but not widely available paper, and is applicable to functions whose values and derivatives can be computed for an arbitrary argument-vector $y = (y_1,\ldots.,y_n)$. Like Davidon's original method, the DFP algorithm will converge exactly to the minimum in n iterations if the function f is exactly quadratic; consequently, rapid convergence may be expected in the neighborhood of a local minimum.

The DFP method uses local information only, and therefore could converge to a local optimum that was not the global optimum. Unless we can assure ourselves (e.g., by a rigorous proof) that there is no spurious local optimum for the particular problem at hand, we must try a number of starting points for our iterative solution process to be reasonably confident that we have found the true solution.

Our problem is to select the group retention rates in such a way as to minimize the ratio of total cost to total utility of the enlisted force, when the assumed costs of changing those group retention rates are included along with training costs, pension costs, and direct pay and allowances. For simplicity we will denote these group retention rates (GRR) by y_1, $y_2,\ldots.y_n$. (In fact, we expect n to be about 6, y_1 to be the average retention rate in LOS years 1-3; y_2, in year 4; y_3, in years 5-9; y_4, in years 10-18; y_5 in years 19-20; and y_6 in years 21-30.) We will refer to the function to be minimized as $f(y) = f(y_1,\ldots.,y_n)$, and that is simply the constrained minimum cost/utility ratio as determined in the preceding section. So we are employing the LDP process as a slave to the DFP optimization of f. For simplicity, we suppress the subscripts and refer simply to the vector y of GRR.

The essence of the DFP algorithm is to proceed iteratively from an arbitrary starting point y^o, via a sequence of points $y^1, y^2, \ldots, y^n, \ldots$ until a point is reached that appears to be a local minimum of f.

Generally we will have $f(y^k) < f(y^{k-1})$, so that the function value will be reduced at each step. Each iterative step consists of calculating a direction in the argument space and then choosing (by some one-dimensional search process) the length of step to be taken so that the minimum of f on that line is found. This subsidiary minimization process typically requires from three to a dozen evaluations of function f, but need not require any evaluation of the gradient vector:

$$g = (g_1, \ldots, g_n) = \left[\frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial y_2}, \ldots, \frac{\partial f}{\partial y_n}\right]. \tag{4}$$

Especially when n is large, evaluation of the gradient g is generally very slow in comparison with evaluation of the function f.

The direction s^k of the k^{th} iterative step is calculated from the gradient by applying a definite matrix (negative-definite for minimization):

$$H^k = (H_{\alpha\beta}{}^k) \tag{5}$$

to obtain the "modified gradient"

$$s^k = H^k g^k. \tag{6}$$

The matrix H, whose negative was called by Davidon the "variable metric," is modified after each iterative step in such a way as to preserve its negative-definite property, while causing the new direction to have suitable orthogonality properties with respect to previous directions.

(For example, $H^O = -I$ makes the first step a "steepest-descent" step.) Those orthogonality properties, if f happened to be exactly quadratic, would cause the matrix H to converge to the constant matrix $-G^{-1}$ in at most n steps, where

$$G = (G_{\beta\lambda}) = \left[\frac{\partial^2 f}{\partial y_\beta \, \partial y_\lambda}\right]$$

is the constant matrix of second derivatives of f. If f is not quadratic, G will not be constant; nevertheless, we may hope that H will approach $-G^{-1}$, evaluated at the minimum of f.

All modified-gradient methods share the above properties; see Powell (1970) for a masterful summary. At each stage, a new matrix H^{k+1} is computed from the previous H^k in some definite way, generally using the vectors

$$\nu^k = (\nu_\beta{}^k) = g^{k+1} - g^k \tag{8}$$

and

$$\sigma^k = (\sigma_\beta{}^k) = y^{k+1} - y^k. \tag{9}$$

The DFP method computes H^{k+1} from H^k by the relation[1]

$$H^{k+1} = H^k - \frac{(\sigma^k)\,(\sigma^k)'}{(\sigma^k)'\,(\nu^k)} - \frac{(H^k\nu^k)\,(H^k\nu^k)'}{(\nu^k)'\,H^k\,(\nu^k)}\ . \tag{10}$$

Fletcher and Powell (1963) shows[2] that, if the function f is exactly quadratic in y and possesses a unique minimum (in which case G is a positive definite), H^n will be exactly the constant matrix $-G^{-1}$, and $s^n = H^n g^n$ will generate exactly the desired step to the minimum of f. Because a sufficiently smooth function, in the neighborhood of its minimum, can often be approximately represented by a quadratic, we can expect good performance from the DFP algorithm once it gets close enough to the minimum. Successful applications have been reported in the literature.

---

[1]Column vectors are denoted by the symbols ν^k, σ^k, and the transposition operator is described by ( )′ .

[2]Our matrix H and the matrix H of Powell (1970) correspond to the negative of the matrix H of Fletcher and Powell (1963). Contrary to the statement in Powell (1970) after equation (30), the above equation (10) is applicable without change whether f is to be minimized or maximized. The difference in sign between equation (30) of Powell (1970) and equation (7) of Fletcher and Powell (1963) is due to the difference in defining H.

Note that the DFP algorithm requires at each step a linear search for a minimum. Explicitly,

$$f(y^k + \alpha s^k),$$

considered as a function of the real variable $\alpha \geq 0$ (which can always be done), should be minimized. Box (1966) has already pointed out that this subsidiary linear-search problem may be a serious weakness because there may be lines along which no minimum exists, even when the function f possesses a well-behaved and unique minimum. We do not anticipate difficulties of this sort in our proposed application, but must await actual experience before we can give a conclusive report.

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